The Joint Dynamics of Labour and Capital

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Joint Dynamics of Firms' Labour and Capital

- Firms make a joint decision
- Empirical evidence of frictions in adjusting both
- Implications of joint dynamics
  1. Frictions of one slow adjustment of the other
  2. Joint distribution
  3. Degree of substitutability between factors matters

What We Do and What We Find

- Analyse joint empirical dynamics of labour and capital
    - Find more correlation across factors than autocorrelation
- Extend lumpy investment models to feature
    - Non-Unitary Elasticity of Substitution between Labour and Capital
    - Adjustment Frictions on Labour
- Improve fit of correlation of firm level investment with lagged adjustment
- Better match probability of adjustment conditional on hiring

Outline

1. Empirical Joint Dynamics
2. Model Description
3. Partial Equilibrium Responses
4. Model Moments versus Empirical Moments
5. Next Steps

Empirical Joint Dynamics

Lagged Investment puzzle

- Well established fact that lagged investment is predictive of future investment
- Difficult for lumpy investment models to match
- Does hiring have predictive power?
- What predicts hiring?

Dependent Variable:	Real Investment to Capital		
Model:	(1)	(2)	(3)
Lagged Real Investment to Capital	0.1472***	0.1329***	0.0120**
	(0.0051)	(0.0050)	(0.0055)
Lagged Hiring to Labour	0.1426***	0.1214***	0.0853***
	(0.0052)	(0.0051)	(0.0054)
Controls			
Q and Cash to Assets		Yes	Yes
Fixed-effects			
SIC by Year			Yes
Company			Yes
Fit statistics			
Observations	133,715	133,715	133,715
R^2	0.06549	0.09680	0.34900

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Dependent Variable:	Hiring to Labour		
Model:	(1)	(2)	(3)
Lagged Real Investment to Capital	0.0890***	0.0778***	0.0488***
	(0.0045)	(0.0044)	(0.0049)
Lagged Hiring to Labour	0.0843***	0.0674***	-0.0493***
	(0.0051)	(0.0050)	(0.0055)
Controls			
Q and Cash to Assets		Yes	Yes
Fixed-effects			
SIC by Year			Yes
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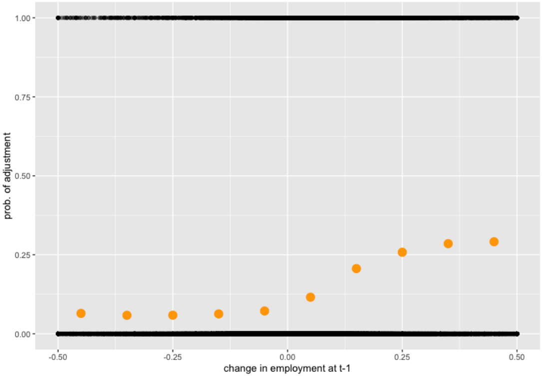
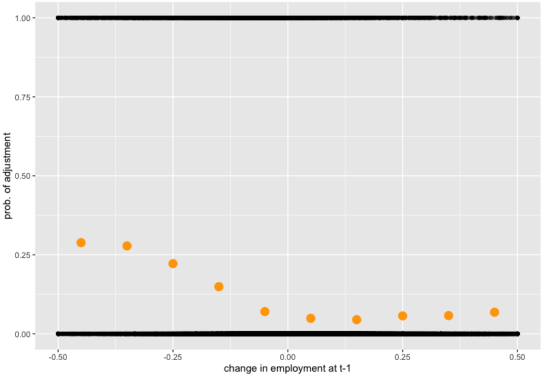
Shape of Adjustment Probability

- Think about extensive versus intensive margin.
- Conditional on past adjustment what proportion of firms adjust?
- Is there asymmetry in predictive power?

Capital Adjustment Probability Conditional on Employment Change



(a) Prob. of adjusting up and lagged emp change



(b) Prob. of adjusting down and lagged emp change

Model Description

Outline of Firm Model

Continuum of firms who choose capital k and labour l to maximise the net present value of profits

$$V(e_0, k_0, l_0) = \max_{\{k_t\}_{t=1}^{\infty}, \{l_t\}_{t=1}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{1}{1+r_t} \left[e_t F(k_t, l_t) - w_t l_t - AC(k_t, k_{t+1}, l_t, l_{t+1}) \right]$$

- Idiosyncratic productivity e_t follows a $AR(1)$ with normal innovations
- F CES with elasticity of substitution ρ and returns to scale parameter α

$$F(k_t, l_t) = \left(\omega l_t^{\frac{\rho-1}{\rho}} + (1-\omega) k_t^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho\alpha}{\rho-1}}$$

Unpacking Adjustment Cost Function

$$AC(k_t, k_{t+1}, l_t, l_{t+1}))$$

Multiple forces lead to adjustment

1. Depreciation of capital at rate δ_k
2. Attrition of workers at rate δ_l
3. Changes in idiosyncratic productivity

Unpacking Adjustment Cost Function

$$
\begin{aligned}
AC(k_t, k_{t+1}, l_t, l_{t+1})) =& w\xi \mathbb{1}(k_{t+1} \neq (1-\delta_k)k_t) \\
&+ p[1-(1-\gamma)\mathbb{1}(k_{t+1} \leq (1-\delta_k)k_t)](k_{t+1}-(1-\delta_k)k) \\
&+ \frac{\chi}{2}\left(\frac{k_{t+1}-(1-\delta_k)k_t}{k_t}\right)^2 k_t + \frac{\phi}{2}\left(\frac{l_{t+1}-(1-\delta_l)l_t}{l_t}\right)^2 l_t
\end{aligned}
$$

Firms face various adjustment costs

1. Fixed capital adjustment costs ξ
2. Partial irreversibility of capital γ
3. Convex costs in both capital χ and labour ϕ

Fixed Adjustment Cost (ξ)

$$
\begin{aligned}
AC(k_t, k_{t+1}, l_t, l_{t+1})) =& w\xi \mathbb{1}(k_{t+1} \neq (1-\delta_k)k_t) \\
&+ p[1-(1-\gamma)\mathbb{1}(k_{t+1} \leq (1-\delta_k)k_t)](k_{t+1}-(1-\delta_k)k) \\
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\end{aligned}
$$

- Each firm draws at start of each period iid from distribution G with mean μ
- Cost is paid in units of labour
- Long tail of firm investment Empirical Dist

Irreversibility (γ)

$$\begin{aligned} AC(k_t, k_{t+1}, l_t, l_{t+1})) =& w\xi\mathbb{1}(k_{t+1} \neq (1-\delta_k)k_t) \\ &+ p[1-(1-\gamma)\mathbb{1}(k_{t+1} \leq (1-\delta_k)k_t)](k_{t+1}-(1-\delta_k)k) \\ &+ \frac{\chi}{2}\left(\frac{k_{t+1}-(1-\delta_k)k_t}{k_t}\right)^2 k_t + \frac{\phi}{2}\left(\frac{l_{t+1}-(1-\delta_l)l_t}{l_t}\right)^2 l_t \end{aligned}$$

- Motivated by evidence of specificity of capital
- Buy capital at price p
- Sell at γp, $\gamma \leq 1$
- Reduces large investments

Quadratic Capital Adjustment Costs (χ)

$$\begin{aligned} AC(k_t, k_{t+1}, l_t, l_{t+1})) = & w\xi \mathbb{1}(k_{t+1} \neq (1-\delta_k)k_t) \\ & + p[1-(1-\gamma)\mathbb{1}(k_{t+1} \leq (1-\delta_k)k_t)](k_{t+1}-(1-\delta_k)k) \\ & + \frac{\chi}{2}\left(\frac{k_{t+1}-(1-\delta_k)k_t}{k_t}\right)^2 k_t + \frac{\phi}{2}\left(\frac{l_{t+1}-(1-\delta_l)l_t}{l_t}\right)^2 l_t \end{aligned}$$

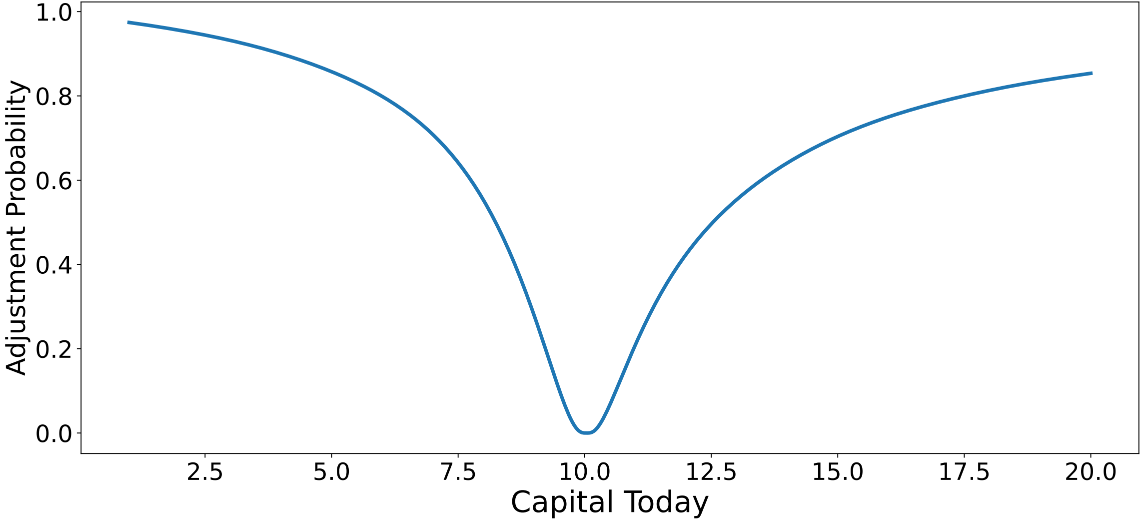
- Used by Winberry (2019)
- Also reduces large investments

Quadratic Labour Adjustment Costs (ϕ)

$$
\begin{aligned}
AC(k_t, k_{t+1}, l_t, l_{t+1})) =& w\xi \mathbb{1}(k_{t+1} \neq (1-\delta_k)k_t) \\
&+ p[1-(1-\gamma)\mathbb{1}(k_{t+1} \leq (1-\delta_k)k_t)](k_{t+1}-(1-\delta_k)k) \\
&+ \frac{\chi}{2}\left(\frac{k_{t+1}-(1-\delta_k)k_t}{k_t}\right)^2 k_t + \frac{\phi}{2}\left(\frac{l_{t+1}-(1-\delta_l)l_t}{l_t}\right)^2 l_t
\end{aligned}
$$

- Makes labour a slow moving stock
- Draws out response of MPK to a productivity shock
- Labour hoarding literature uses asymmetric cost

Adjustment Probability Before Fixed Cost Draw Without Irreversibility



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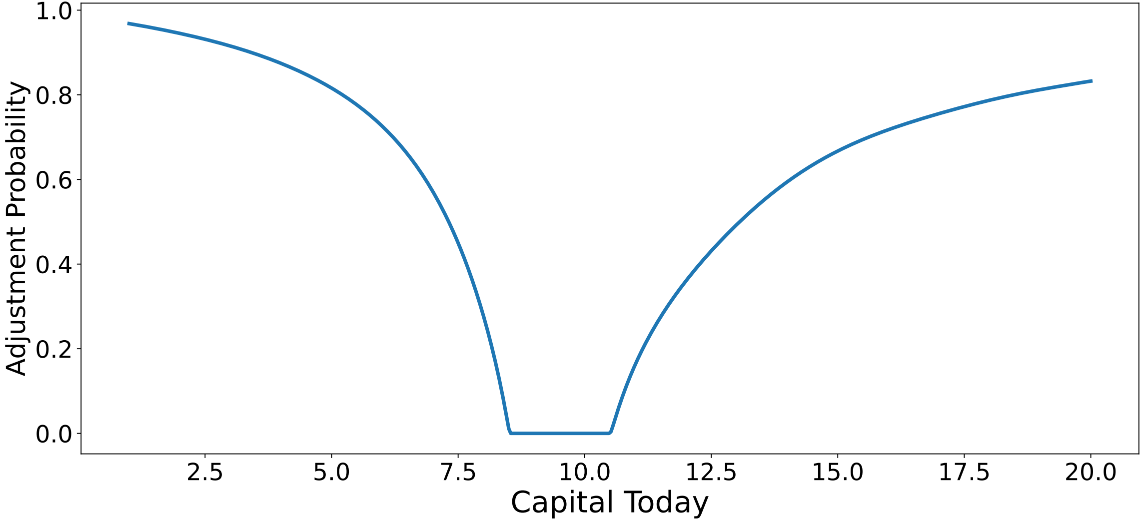

Other models within this framework

$$\rho = \text{Elasticity of Substitution},\ \gamma = \text{Irreversibility of Capital},$$
$$\chi = \text{Convex Capital Costs},\ \phi = \text{Convex Labour Costs}$$

- Khan Thomas

$$\rho = 1, \gamma = 1, \chi = 0, \phi = 0$$

- Winberry

$$\rho = 1, \gamma = 1, \chi = 2.950, \phi = 0$$

- Our favoured parameters

$$\rho < 1, \gamma < 1, \chi = 0, \phi > 0$$

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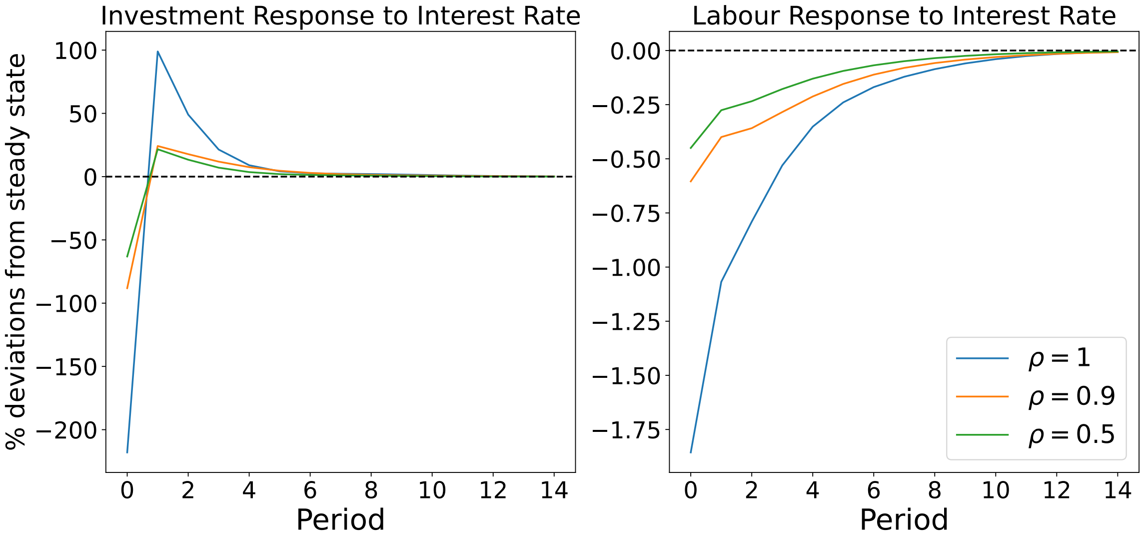
Next: How do these parameters change the responses of the model?

Partial Equilibrium Responses

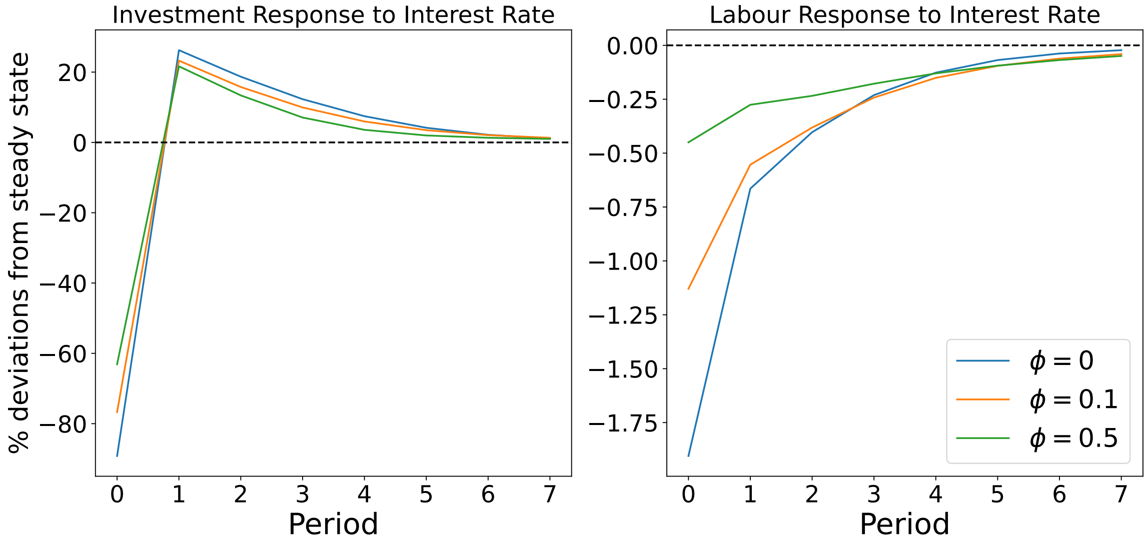
Partial Equilibrium Responses of Firms

- Use method of Auclert et al (2020) to calculate Jacobians of firm block
- Study investment and labour response to interest rates
- How does complementarity matter?
- Explore role of different adjustment frictions

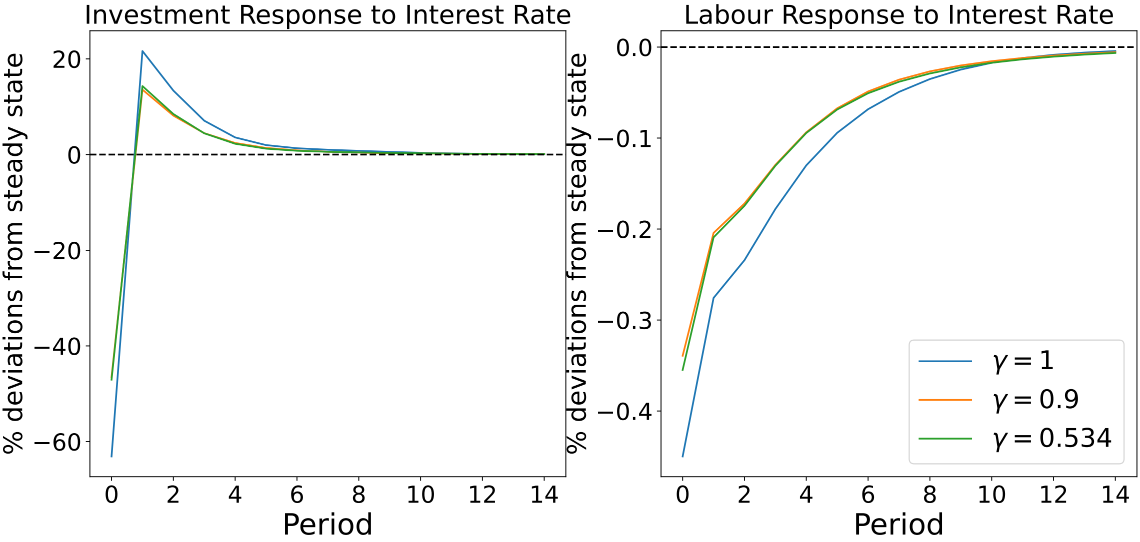
Complementarity of Labour and Capital Greatly Dampens Response



Labour Adjustment Costs Dampen Both Responses

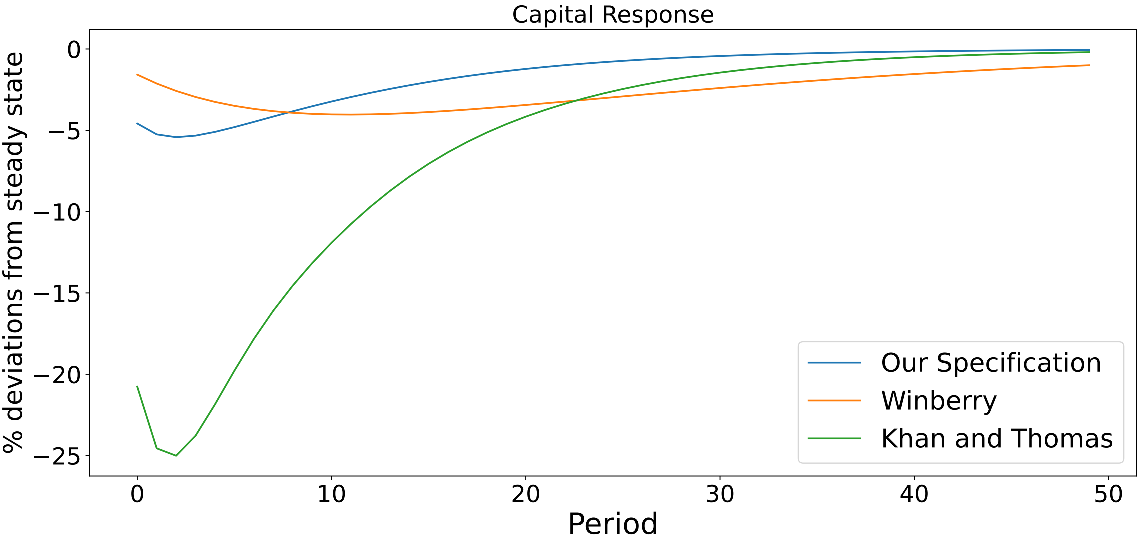


Irreversibility further Dampens Response



Convex Costs Implies Long Response

K/L Ratio



Summing Up

- Most extreme results driven by Cobb-Douglas ($\rho = 1$)
- Labour adjustment costs do reduce sensitivity
- Can get reasonable response without convex costs using irreversibility
- Convex capital costs imply very drawn out stock dynamics
- Our parameters get reasonable investment distribution Dists

Summing Up

- Most extreme results driven by Cobb-Douglas ($\rho = 1$)
- Labour adjustment costs do reduce sensitivity
- Can get reasonable response without convex costs using irreversibility
- Convex capital costs imply very drawn out stock dynamics
- Our parameters get reasonable investment distribution Dists

Next: Which models match our empirical moments?

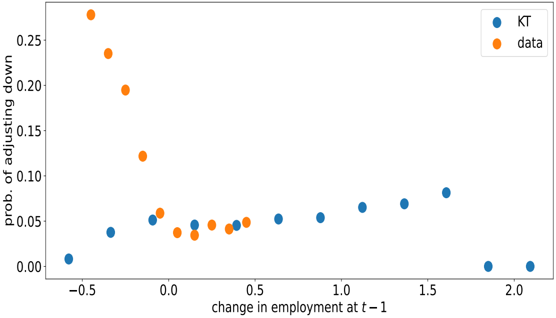
Model Moments versus Empirical Moments

How well are the moments matched?

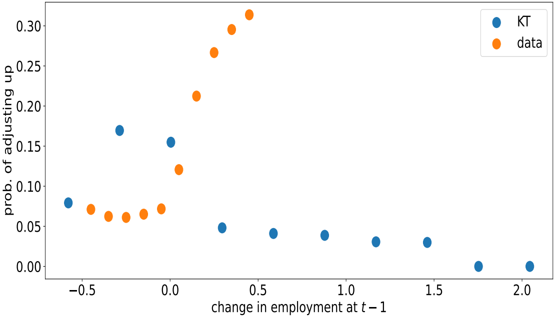
We look at several aspects of the joint movement of labour and capital.

- Look at predictive power of employment growth for capital growth
- Can we also generate autocorrelation of investment?

Khan and Thomas $\rho = 1, \phi = 0, \chi = 0$

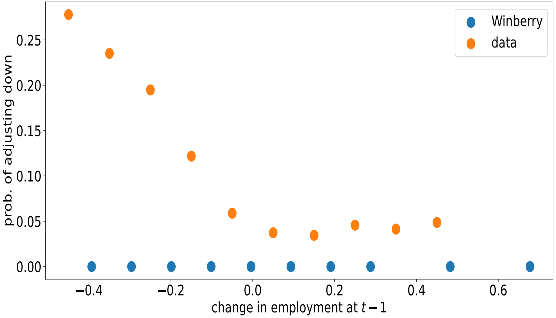


(a)



(b)

Winberry $\rho = 1, \phi = 0, \chi = 2.950$



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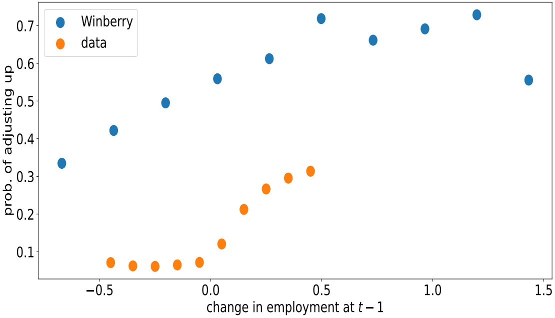
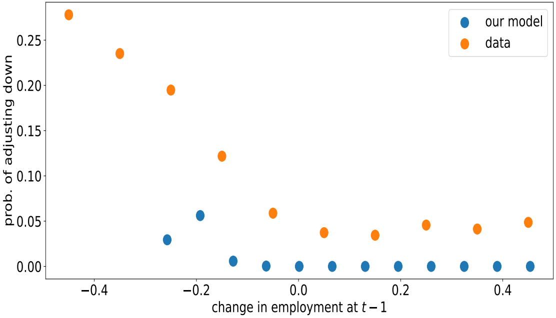


(b)

Our preferred specification $\rho = 0.5, \phi = 0.1, \chi = 0$



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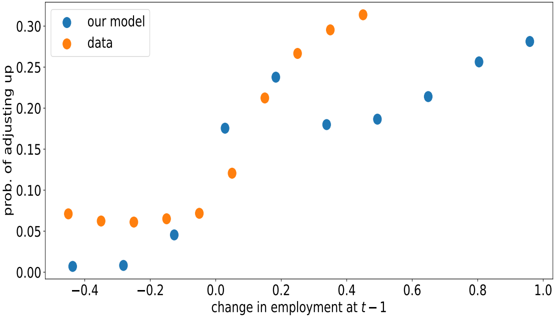
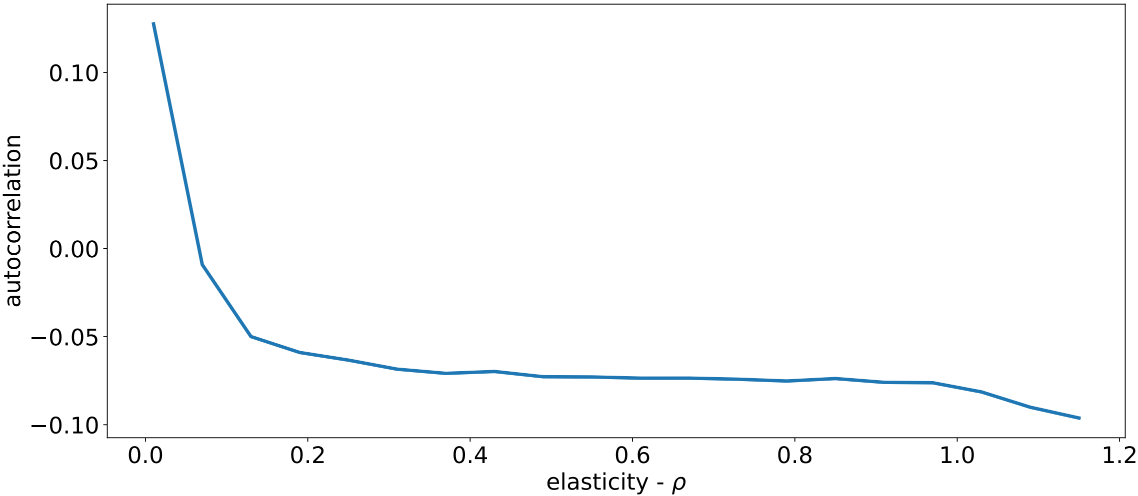
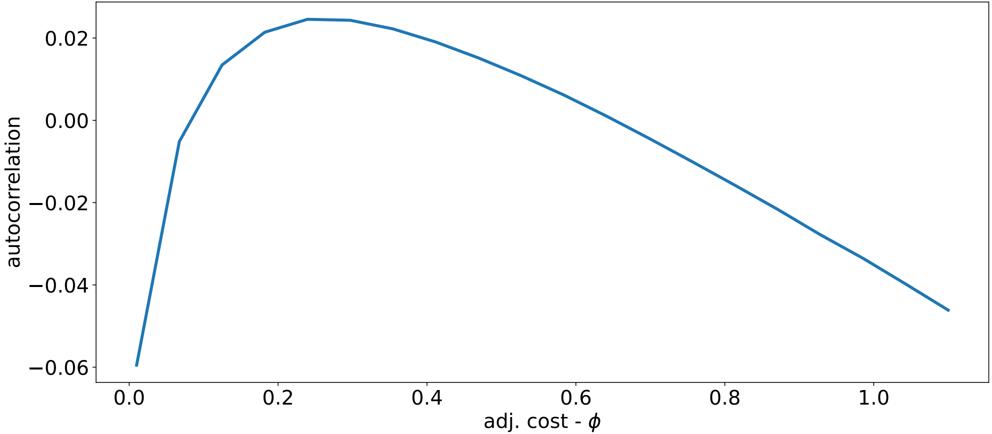


(b)

More Complementarity Higher Auto-Correlation



Small Labour Adjustment Costs Can Generate Positive Auto-Correlation

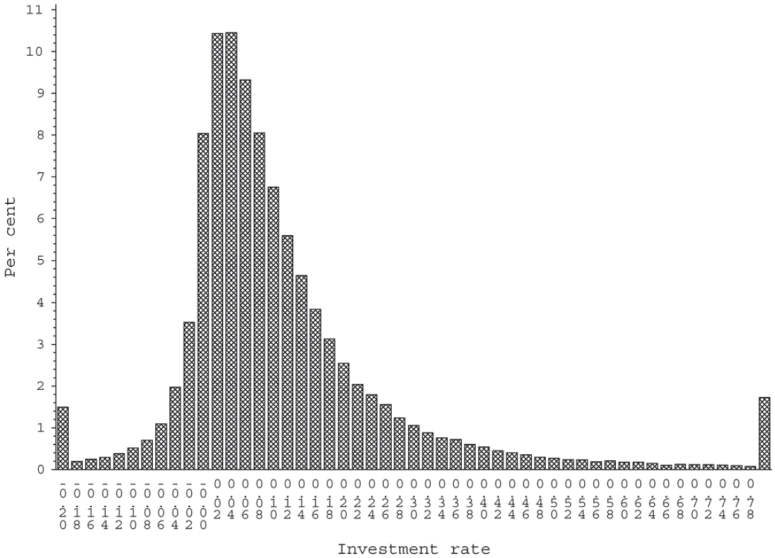


Takeaways and Next Steps

- Takeaways
  - Complementarity of factors indirectly supported
  - Irreversibility as well
  - Labour adjusts slowly in data
  - Convex capital costs implies implausibly long transitions
  - Can generate positively autocorrelated investment
- Next Steps
  - Alternate labour adjustment costs
  - Chilian data?

Empirical Investment Distribution

Fixed Adjustment Costs | Summing up



Investment Distribution No Irreversibility

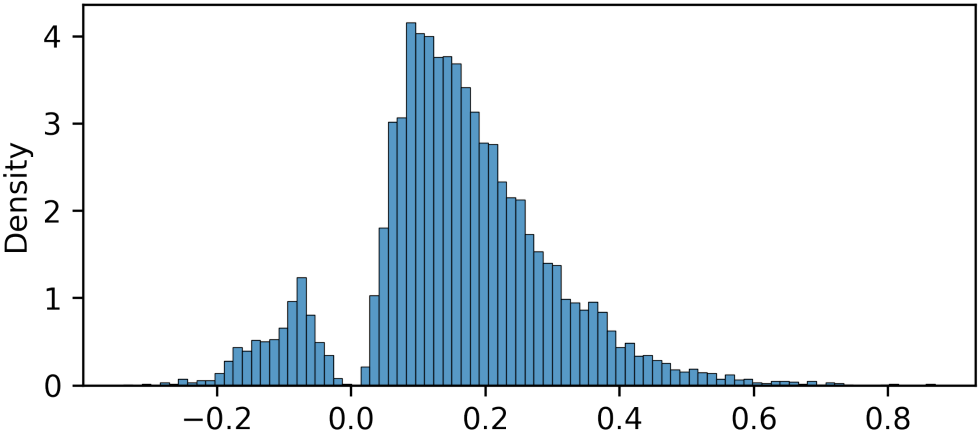


Figure: Histogram of Investment Back

Investment Distribution Irreversibility

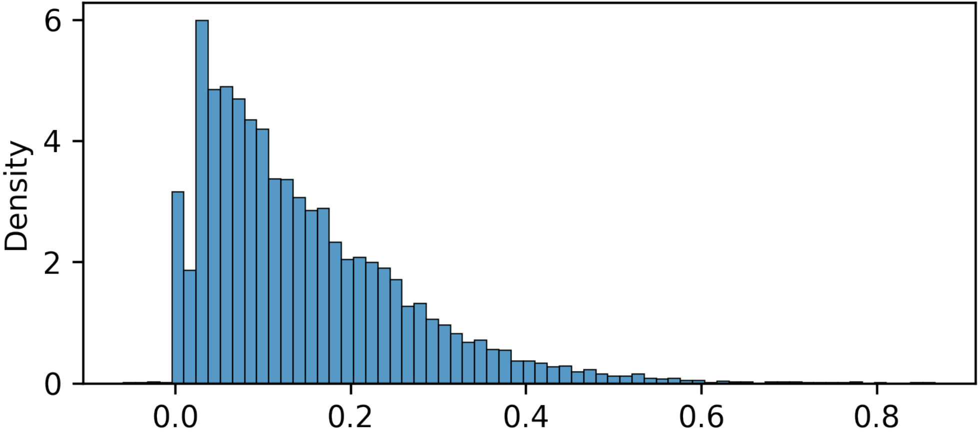
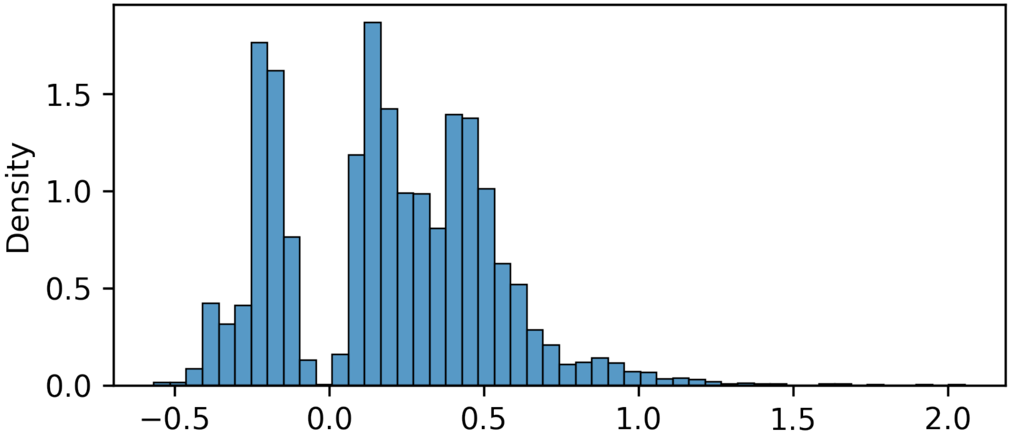


Figure: Histogram of Investment Back

Investment Distribution Khan Thomas



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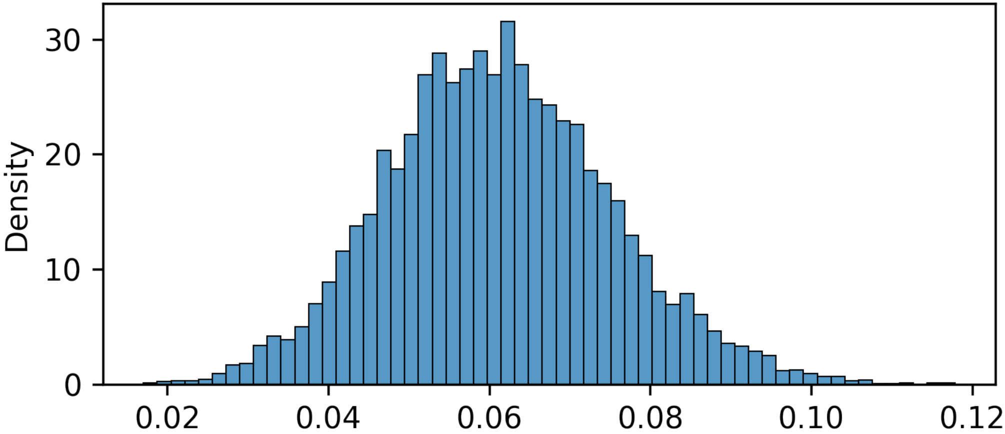
Investment Distribution Winberry



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