Tariffs, Trade Deficits, and Liquidity Supply *

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Abstract

We use a simple model to study the effects of tariffs on the current account balance when a country runs permanent trade deficits. The model features a country that supplies a liquid asset to the rest of the world, which allows the country to run a permanent and sustainable current account deficit. Unlike in models based on the traditional intertemporal approach, tariffs do not affect saving through their effects on real interest rates, and have muted effects on the current account balance. We also show how shifts in liquidity demand and taxes on world liquidity can be interpreted from a trade-policy perspective.

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1 Introduction

The recent policy debate on tariffs and trade wars has revived interest in the connection between tariffs and the current account balance, given that a proclaimed objective of the Trump administration is to reduce the U.S. trade deficit.[1]

There is a traditional argument that is often used in macroeconomic discussions on the effect of tariffs. The current account identity implies that tariffs can reduce the trade deficit only if they induce a change in the net savings of the country affected. If we believe that net savings and more broadly international capital flows are determined by forces that are relatively insensitive to trade policy, then higher tariffs will not produce a significant reduction of a country's trade deficit.

The traditional argument above relies on some notion of separation between the financial side and the trade side of the current account. In modern micro-founded open-economy models, however, this separation does not generally hold, as all prices are endogenous and a change in the tariff regime generally affects prices within periods, across periods, and across states of the world. The question, then, is: through which mechanisms do tariffs affect the financial side?

In this paper, we focus on two channels through which tariffs affect saving and investment decisions: how tariffs affect the real interest rate faced by domestic and foreign consumers, and how they affect the demand for liquidity of domestic and foreign consumers.

The first contribution of the paper is to unpack a mechanism that is present in many existing models: higher tariffs increase the real interest rate faced by domestic consumers, leading to higher domestic saving, and a lower trade balance. We show that for this mechanism to operate it is crucial that the country is running a *transitory* trade deficit. To make this point, we contrast the implications of two models that have identical trade structures (a standard two-good Armington model) but differ in the intertemporal dimension.

[1] Here we are interested in the effect of tariffs on the overall deficit. The Trump administration has also formulated its objectives in terms of reducing bilateral trade deficits. We do not discuss that connection here.

The first model is a two period model in the tradition of the standard intertemporal approach to the current account. In that model, trade imbalances are by necessity transitory, and tariffs affect saving incentives because they have asymmetric effects on the relative prices of goods across periods. We show that this mechanism works whether tariffs are imposed unilaterally or bilaterally. In fact, if the Foreign country retaliates with its own tariffs, the reduction in the Home trade deficit is amplified.

The second model is a two-country model in which one country acts as a world liquidity supplier. Countries hold gross asset and liability positions against each other. The liabilities issued by one country have a special nature: they can be used as world liquidity. That country earns a higher return on its assets than on its liabilities. The model is consistent with an interpretation of the U.S. current account deficits formulated in work by Gourinchas and Rey (2007), Caballero et al. (2008), and Maggiori (2013).

In this model, trade imbalances can persist indefinitely and therefore tariffs have more muted effects, as they may not affect the real interest rate. In the simplest version of the model, where all assets are denominated in the creditor's currency, we have a proposition that exactly reflects the traditional argument summarized above: tariffs have no effect on the trade deficit, as in that model we have complete separation between the financial side and the trade side. With more realistic currency denomination, valuation effects can reduce the deficit. But unlike the intertemporal model, foreign retaliation reverses the effect, pushing the trade deficit back towards its initial level.

Our second contribution is to use this framework to highlight a tension between reducing trade deficits and improving domestic welfare. We show that the valuation effects that allow tariffs to reduce the deficit also tend to lower domestic welfare. This tension reappears when we consider other policies, such as a tax on foreign holdings of liquid bonds. Policies that successfully reduce the trade deficit often come at the expense of domestic welfare. We call this tradeoff "Miran's dilemma".

A number of papers have explored the effect of trade costs on the trading of assets across borders, including Fitzgerald (2012), Eaton et al. (2016) and Reyes-Heroles (2016). The closest paper is Reyes-Heroles (2016), which provides a quantitative exploration of the intertemporal channel. All these papers are crucially calibrated by assuming that a period in which the borrowing country receives a net transfer from the rest of the world is eventually followed by a period in which the net transfer is reversed, so they feature some version of the mechanism we describe in Section 2, which is muted in our model of Section 3.

Following the introduction of tariffs by the second Trump administration, several important papers have emerged examining the connection between tariffs and trade deficits. Aguiar et al. (2025) explore how balanced trade can be reconciled with pre-existing non-zero net foreign asset positions. They show that endogenous terms-of-trade effects can ensure balanced trade while satisfying financial obligations, a mechanism related to the valuation effects in Section 3. In a closely related contribution, Itskhoki and Mukhin (2025) argue that tariffs cannot close long-run trade deficits without valuation effects, a conclusion that is in line with our model of permanent deficits where the real interest rate channel is muted. In a model with exogenous terms of trade, Costinot and Werning (2025) show that whether tariffs reduce trade deficits depends on a single sufficient statistic: the slope of Engel curves in the space of imports and exports. However, they abstract from valuation effects, which are central to our analysis.

2 The Intertemporal Argument

In this section, we revisit the intertemporal argument that trade frictions, and, in particular tariffs, can temper current-account imbalances. We derive our results using a simple Armington (1969) model of world trade with two countries and two goods.

Relative to the analysis in Obstfeld and Rogoff (2000) mentioned in the introduction, we have tariffs instead of transportation costs and, more importantly, we consider two large economies. As we shall see, this introduces general equilibrium forces. We show that these forces tend to go in the same direction as the mechanism discussed in that paper and allow us to consider also the effect of retaliatory tariffs by the foreign country.

There are two countries, called Home and Foreign, and two goods, labeled H and F. There are two periods $t = 1, 2$. In each period, consumers in each country receive a deterministic endowment of the goods and are perfectly specialized. Namely, Home consumers receive the endowment Y_{Ht} of good H and the Foreign consumer receives Y^*_{Ft} of good F. Following the common convention, asterisks will be used to denote Foreign variables.

The preferences of the home consumer are represented by the utility function

$$u(C_1) + \beta u(C_2),$$

where $u(C_t) = C_t^{1-\gamma}/(1-\gamma)$ and the consumption aggregate in each period is

$$C_t = \left(\omega^{\frac{1}{\epsilon}} C_{Ht}^{1-\frac{1}{\epsilon}} + (1-\omega)^{\frac{1}{\epsilon}} C_{Ft}^{1-\frac{1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{1}$$

where C_{Ht} and C_{Ft} are consumption of the two goods. The foreign consumer has similar preferences, except that in the definition of C^*_t the roles of the two goods are inverted, so

$$C^*_t = \left(\omega^{\frac{1}{\epsilon}} \left(C^*_{Ft}\right)^{1-\frac{1}{\epsilon}} + (1-\omega)^{\frac{1}{\epsilon}} \left(C^*_{Ht}\right)^{1-\frac{1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}.$$

We assume $\epsilon \geq 1$, consistently with common calibrations. Also, in the direction of realism, we assume that each country's preferences are biased towards the domestically produced good, that is, $\omega > 1/2$.[2]

Consumers enter period $t = 1$ with a zero initial financial position, borrow and lend on the world capital market at the interest rate i_1 and, within each period, trade home and foreign goods at the prices P_{Ht} and P_{Ft}. All prices are denominated in a common unit of account.

2.1 Static Equilibrium

We begin by analyzing the goods market equilibrium within each period t, for a given level of the Home trade deficit.

The period t budget constraint of the home consumer is

$$P_{Ht}C_{Ht} + (1+\tau_t)P_{Ft}C_{Ft} = P_{Ht}Y_{Ht} + T_t + P_{Ht}D_t,$$

[2] We omit transportation costs. However, with perfect specialization, one can show that a model with iceberg costs is structurally equivalent to the model here, with transportation costs embedded in the parameter ω.

where τ_t is an ad valorem tariff on the imported F good, T_t is a lump sum transfer used by the domestic government to rebate the tariff receipts, and D_t is the home country trade deficit denominated in home goods. An analogous budget constraint holds for the foreign consumer, with τ_t^* denoting the foreign tariff on the H good.

Combining the consumer and government budget constraint we rewrite the trade deficit equation as:

$$\underbrace{P_{Ft}C_{Ft}}_{\text{imports}} - \underbrace{P_{Ht}(Y_{Ht} - C_{Ht})}_{\text{exports}} = P_{Ht}D_t. \tag{2}$$

A symmetric equation can be derived for the foreign consumer.

Combining the trade balance condition above and consumer optimality, we can then derive the demand for good H of the Home consumer, as a function of the relative price P_H/P_F, the home trade deficit D_t, and the tariff τ. We denote this demand function

$$C_{Ht} = \mathcal{C}_H(P_{Ht}/P_{Ft}, D_t, \tau),$$

and provide the explicit formula in the appendix. In the same way, we derive the demand for good H of the foreign consumer.

We can then write the goods market clearing condition as

$$\mathcal{C}_H(P_{Ht}/P_{Ft}, D_t, \tau) + \mathcal{C}_H^*(P_{Ht}/P_{Ft}, D_t, \tau^*) = Y_{Ht}. \tag{3}$$

This equation gives us the equilibrium terms of trade (i.e., the relative price P_H/P_F for any possible level of the trade deficit D_t.[3]

Standard comparative statics show that, for given D_t, an increase in the tariff τ increases the domestic terms of trade P_{Ht}/P_{Ft}, while an increase in the tariff τ_t^* reduces them, as expected. However, the crucial observation for the analysis to follow is that the effect of the tariffs τ and τ^* on terms of trade depends in general on the level of D_t. In the following proposition we show that, all else equal, a higher level of the trade deficit amplifies the effect of the tariff on the terms of trade.

Omitting time subscripts, let us write the relation between trade deficit, tariffs, and the equilibrium terms of trade, as

$$\frac{P_H}{P_F} = \rho(D, \tau, \tau^*).$$

[3] By Walras' law, equilibrium in the market for good F is also satisfied.

We then have the following result. The proof is in Appendix A.1.

Proposition 1 *(Amplification of TOT effects) In the two good Armington model with $\epsilon > 1$ and $\omega > 1/2$, the effect of increasing the tariff τ on the relative price P_H/P_F is increasing in the level of the trade deficit D:*

$$\frac{\partial^2 \rho(D, \tau, \tau^*)}{\partial \tau \partial D} > 0.$$

The intuition for this result is natural: when D is larger the domestic economy represents a larger fraction of world demand for good H and the tariff has a larger proportional effect on total demand, and a larger price response is needed to bring the market back to equilibrium. An analogous result also shows that the effect of τ^* (which is to reduce P_H/P_F) is *dampened* by a larger trade deficit.

This effect is at the basis of the intertemporal results to which we now turn.

2.2 Dynamic Equilibrium

The Euler equation of the domestic consumer is

$$u'(C_1) = (1 + r_1)\beta u'(C_2),$$

where r_1 is the real interest rate

$$1 + r_1 = (1 + i_1)\frac{P_1}{P_2},$$

and P_t is the consumer price index

$$P_t = \left(\omega P_{Ht}^{1-\epsilon} + (1-\omega)((1+\tau_t)P_{Ft})^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}. \tag{4}$$

The Euler equation of the foreign consumers is analogous.

The intertemporal budget constraint for the home country can be written in terms of trade deficits in the two periods as

$$\frac{P_{H1}}{P_1}D_1 + \frac{1}{1+r_1}\frac{P_{H2}}{P_2}D_2 = 0. \tag{5}$$

Combining the two Euler equations and rearranging gives the condition

$$\frac{u'\left(C_1\right)}{\beta u'\left(C_2\right)} = \left(\frac{P_1/P_1^*}{P_2/P_2^*}\right)\frac{u'\left(C_1^*\right)}{\beta^* u'\left(C_2^*\right)}. \tag{6}$$

The two real rates r_1 and r_1^* are in general different, for two reasons: the two consumers face different relative prices, due to tariffs, and they consume different consumption baskets, due to the home-bias assumption $\omega \neq 1/2$. The gap between the two real interest rates is crucial for the economy response to tariffs, as we will see shortly.

A simple method to find an equilibrium is to check if a pair (D_1, D_2) satisfies all equilibrium conditions using the following algorithm.

In each period t, given D_t, derive the static terms of trade P_{Ht}/P_{Ft} from the good market clearing condition (3). This step also gives us consumption levels of each good, and the consumption indexes C_t and C_t^*. Next, compute the ratios P_t/P_t^* of the price indexes of the two consumers, which are functions of the relative prices P_{Ht}/P_{Ft}. Finally, check two equilibrium conditions: the intertemporal budget constraint (5) and the Euler equations' consistency condition (6).[4]

We will use a graphical representation of conditions (5) and (6) as two curves in the space (D_1, D_2) to represent an equilibrium and see how it changes with higher tariffs.

2.3 The Effect of Tariffs

Suppose that we start in an equilibrium with zero tariffs and, in equilibrium, the home country is a net borrower in the first period, $D_1 > 0$. The domestic government unilaterally introduces a permanent positive tariff $\tau > 0$. What are the effects on the country's borrowing D_1? What are the effects if the foreign country also simultaneously imposes a positive tariff?

Consider a numerical example with preference parameters:

$$\beta = \beta^* = 1, \quad \gamma = 2, \quad \omega = 0.7, \quad \epsilon = 1;$$

[4]To be precise, the intertemporal budget constraint (5) requires knowledge of r_1, so we first rewrite it as:

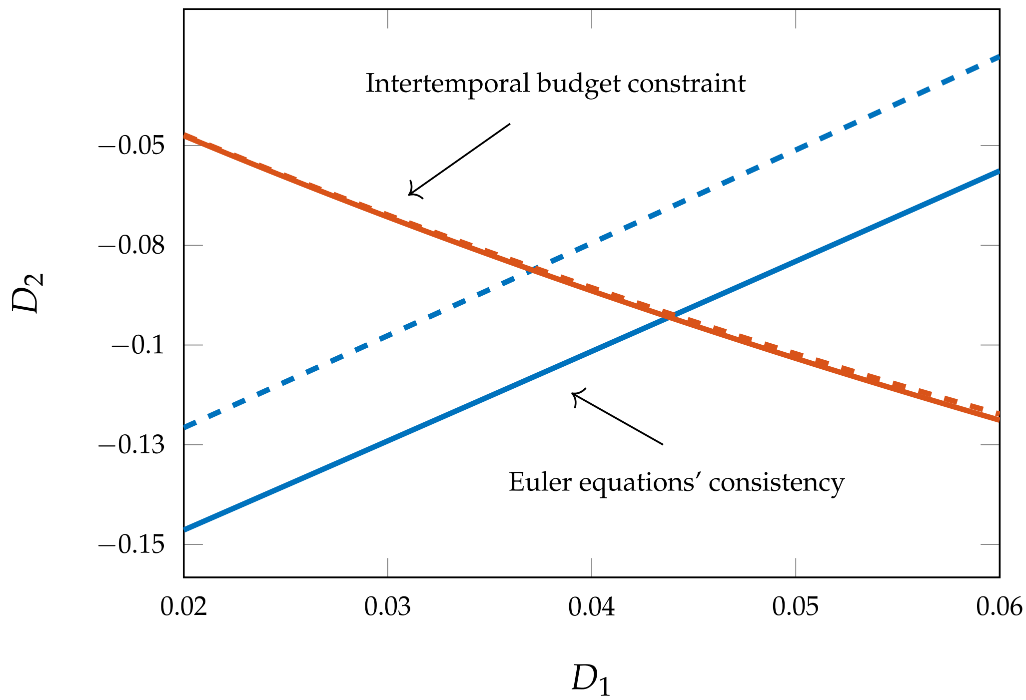
$$\frac{P_{H1}}{P_1}D_1 + \beta\frac{u'(C_2)}{u'(C_1)}\frac{P_{H2}}{P_2}D_2 = 0.$$

and endowments:

$$(Y_{H1}, Y_{H2}) = (1, 2), \qquad (Y_{F1}, Y_{F2}) = (2, 1).$$

In this economy, the home country is a net borrower in equilibrium.

Figure 1: Intertemporal Approach: Comparative Statics



Note: Solid lines: no tariff; dashed lines: $\tau = 0.25$*. See text for model parameters.*

Figure 1 plots the two equilibrium curves (5) and (6) in the space of trade deficit pairs (D_1, D_2). The equilibrium is given by the intersection of the two curves. In the same graph, we show the curves under zero tariffs (solid lines) and with a positive Home tariff $\tau = 0.25$ (dashed lines).

The tariff causes an upward shift of the Euler equations' consistency curve. At the same time, there is a wealth effect that shifts out the intertemporal budget constraint curve, as the tariff improves the terms of trade for the home country. This second effect is very small in our example. Therefore, the shift in the Euler equations' relation dominates and we have a lower equilibrium value of D_1: the home country borrows less in period 1.

Since the shift of the Euler equations' consistency curve is central to the result here, let us analyze it in detail. The logic behind the shift is in the mechanism identified in

Proposition 1. Recall that the effect of a tariff on the terms of trade is amplified when the country is running a larger trade deficit. Since Home runs a deficit in period 1 ($D_1 > 0$) and a surplus in period 2 ($D_2 < 0$), the tariff has a stronger effect on the terms of trade in period 1 than in period 2. This asymmetry is what drives the intertemporal effect.

The tariff affects the equilibrium price indexes through two channels: a direct effect on the cost of imports and an indirect effect through the terms of trade. Both channels work in the same direction. The direct effect raises the home price index when the tariff increases the price of imported goods; this effect is larger when Home spends more on imports, which occurs in period 1 when Home is running a deficit.[5] The indirect effect operates through the goods market: the tariff increases the relative price P_{Ht}/P_{Ft}, which due to home bias raises the home price index relative to the foreign index. By Proposition 1, this effect is stronger in period 1, when Home controls a larger share of world spending.

Combining the direct and indirect effects, the tariff increases the ratio $(P_1/P_1^*)/(P_2/P_2^*)$ in the Euler equations consistency condition (6). This opens a wedge between the real interest rates faced by the two consumers: the real rate rises for Home and falls for Foreign. To reconcile the two Euler equations, home consumers must save more and foreign consumers less, reducing D_1.

The reason why the arguments just given do not add up to a full analytical proof is that the relative price changes also have income effects which complicate the analytical argument. Nonetheless, experimenting with a wide range of empirically plausible parameters shows that the effects shown in Figure 2 are typical. The left panel shows the trade deficit to GDP ratio, D_1/Y_{H1}, for a range of tariff levels: a higher tariff reduces the deficit and the effect is larger if the Foreign country also imposes a tariff. The right panel shows the corresponding wedge between the home and foreign real interest rates: as expected, a higher tariff opens a larger wedge.

Let us add some observations. First, the effect of tariffs on terms of trade is crucial. In particular, a small open economy version of our model with constant terms of trade would deliver no effect of the tariff on the trade deficit.[6] This is because in our model

[5] With $\epsilon = 1$, spending shares are constant and this direct channel is absent.

[6] This is proved formally in Proposition 11 in the Appendix.

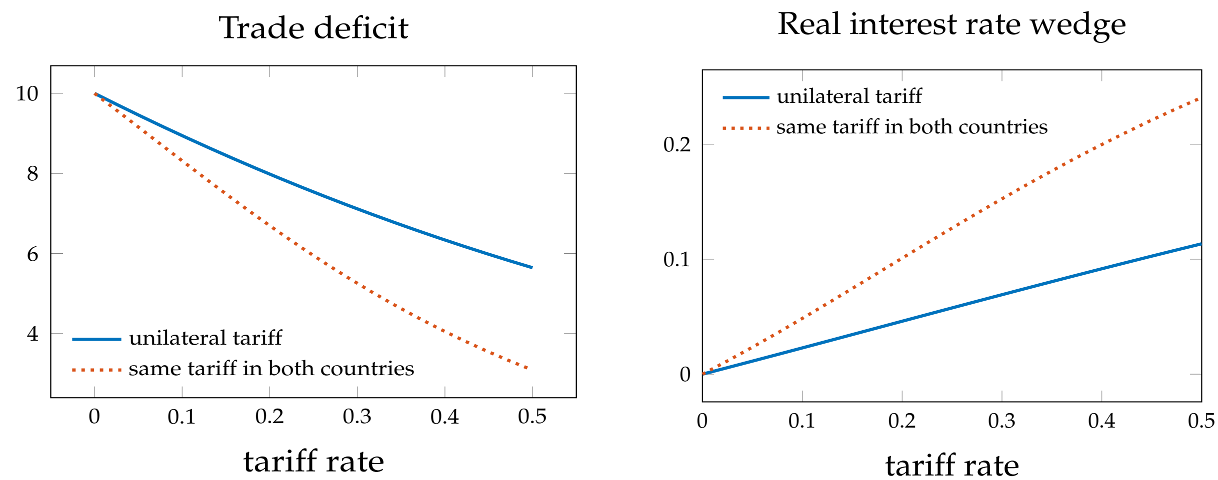


Figure 2: Effect of unilateral and bilateral tariffs on period 1 deficit of home country

Note: Two-country, two-period Armington model with $\beta = \beta^* = 1$, $\gamma = 1$, and $\epsilon = 5$. We calibrate home-bias weights to match spending flows, so Home runs about a 10% trade deficit in the baseline without tariffs.

the set of traded goods and the direction of trade are the same in any equilibrium: Home exports good H and imports good F and this pattern is unaffected by tariffs. See Costinot and Werning (2025) for more on this result.

Second, if the foreign country retaliates by also imposing a tariff, the reduction in the trade deficit is larger. The foreign tariff reduces P_{Ht}/P_{Ft} in both periods, but by the logic of Proposition 1 this effect is lower in period 1 (when Home is running a deficit). This reinforces the asymmetry, opening a larger wedge between the real interest rates and further reducing D_1. The effect can be seen in Figure 2: the bilateral tariff curve (dotted line) is below the unilateral curve (solid line) in the left panel and above it in the right panel.

Third, while the logic above does not immediately translate into a full analytical proof, the following proposition proves the result analytically when trade imbalances are small and driven by differences in discount factors.

Proposition 2 *Consider a family of economies indexed by $\theta \geq 0$. Suppose that:*

(i) θ only enters discount factors: $\beta = \beta(\theta)$ and $\beta^* = \beta^*(\theta)$ with $\beta(0) = \beta^*(0)$;

(ii) at $(\tau, \theta) = (0, 0)$ *the allocation is symmetric (*$D_1 = D_2 = 0$*,* $\rho_1 = \rho_2 = 1$*,* $P_t = P_t^*$*);*

(iii) for $\theta > 0$*,* $D_1(0, \theta) > 0$*,* $D_2(0, \theta) < 0$*, and* $D_t(0, \theta) = O(\theta)$*.*

Then, in a neighbourhood of $(\tau, \theta) = (0, 0)$*, an unexpected permanent increase in the home tariff lowers* D_1*.*

PROOF See Appendix A.3. ■

Summing up, the introduction of a tariff changes intertemporal incentives so as to increase net savings by the country imposing the tariff, thus reducing the trade deficit. The reason is that the tariff affects global spending on different goods with different force depending on the sign of the trade deficit. In the first period, when the trade deficit is positive, the tariff has a stronger effect on the home good price. This increases the real interest rate for the home consumer more than for the foreign consumer, discouraging the home consumer from borrowing. As we shall see, this mechanism is muted in the model of the next section.

3 A MODEL OF WORLD LIQUIDITY SUPPLY

We now introduce a model with an infinite horizon and two types of bonds, liquid and illiquid bonds. Liquid bonds provide liquidity services that are modeled by simply adding them to the utility function.

The model resembles traditional models of money in the utility function, as Sidrauski (1967), with two main differences. First, we interpret broadly the liquid assets issued by the home country as including interest-paying government debt, so liquid bonds pay a nominal interest rate i_b that can be greater than zero. Second, we assume that only the government of Home can issue liquid bonds, capturing the special role that safe, U.S. issued assets play in the world economy and the special position of the U.S. government in emitting these assets.

3.1 Model ingredients

The model is set in continuous time and has an infinite horizon. The preferences of the domestic consumer are

$$\int_0^\infty e^{-\rho t} \left[u\left(C_t\right) + v\left(\frac{B_t}{P_t}\right)\right] dt$$

where the functions u and v are isoelastic

$$u\left(C_t\right) = \frac{1}{1-\gamma} C_t^{1-\gamma}, \quad v\left(\frac{B_t}{P_t}\right) = \frac{\psi^\gamma}{1-\gamma}\left(\frac{B_t}{P_t}\right)^{1-\gamma}.$$

Consumption C_t is a constant elasticity aggregate of consumption of home and foreign goods, as in equation (1) above, and P_t is the consumption price index (4). The domestic budget constraint is

$$P_{Ht}C_{Ht} + (1+\tau_t)P_{FT}C_{Ft} + \dot{B}_t + \dot{A}_t = P_{Ht}Y_{Ht} + i_t A_t + i_{bt}B_t + T_t,$$

where A_t and B_t denote holdings of illiquid and liquid bonds, i_t and i_{bt} are interest rates on illiquid and liquid bonds, τ_t is the tariff, and T_t is a lump-sum transfer to domestic consumers. As in the previous section, all prices are denominated in a common unit of account. The domestic government's budget constraint is

$$\dot{\bar{B}}_t + \tau_t p_{Ft} c_{Ft} = T_t + i_{tb}\bar{B}_t,$$

where $\bar{B}_t$ is the supply of liquid bonds.

The preferences of the foreign consumer are analogous to those of the domestic consumer, with the same discount factor ρ and the same functions u and v, except for the parameter ψ^*, which we allow to be different. As in the model of Section 2, the consumption aggregate C_t^* is symmetric, with the role of the home and foreign good inverted, and all foreign variables are denoted with stars. The foreign government does not issue liquid assets, it just rebates any tariff revenue to the consumers as a lump sum transfer.

3.2 Equilibrium

The static equilibrium conditions are the same as in Section 2.1 and are summarized by the good market equilibrium condition (3) that determines the relative price P_{Ht}/P_{Ft} for a given trade deficit D_t.

Things are different on the financial side, as we now need to ensure equilibrium in the market for both liquid and illiquid bonds. The consumer optimality conditions for illiquid bonds gives the standard Euler equation

$$\gamma \frac{\dot{C}_t}{C_t} = i_t - \pi_t - \rho$$

where inflation, π_t, is the growth rate of the home price index P_t. The following condition ensures an optimal portfolio allocation between liquid and illiquid bonds, considering the marginal trade-off between the extra interest earnings on illiquid bonds and the utility benefit of liquidity

$$(i_t - i_{bt})\, u'(C_t) = v'\left(\frac{B_t}{P_t}\right).$$

The last equation can be rearranged to obtain the demand for liquid balances

$$B_t = \psi \left(i_t - i_{bt}\right)^{-\frac{1}{\gamma}} P_t C_t. \tag{7}$$

This equation has a standard interpretation as a traditional money demand equation: holdings of liquid bonds are proportional to transactions in domestic consumption, and a higher rate-of-return differential $i_t - i_{bt}$ induces agents to economize on liquid bonds. An analogous condition characterizes the foreign demand for liquid bonds. Notice that if i_{bt} approaches i_t there is an unbounded demand for liquid bonds, so, given a finite supply of government bonds $\bar{B}_t$, we must have $i_{bt} < i_t$ in equilibrium. Market clearing in asset markets requires $A_t + A_t^* = 0$ and $B_t + B_t^* = \bar{B}_t$.

As in the two-period model, the trade deficit D_t is the connection between the financial side of the economy and the good markets equilibrium. Adding up the budget constraints of the home consumers and of the home government and using the fact that the net liquid bond position of the country is $\bar{B}_t - B_t = B_t^*$, we obtain

$$P_{Ht} D_t = i_t A_t - i_{bt} B_t^* + \dot{B}_t^* - \dot{A}_t,$$

where the trade deficit D_t is denominated in H goods as in previous sections. A trade deficit can be financed by earning interest income on home assets larger than the interest payments on net home liabilities, or by issuing net home liabilities in excess of purchases of foreign assets.

Define the home net foreign asset position

$$N_t = A_t - B_t^*.$$

The equation above can then be rewritten as

$$P_{Ht} D_t = i_t N_t + (i_t - i_{bt}) B_t^* - \dot{N}_t \tag{8}$$

The second expression on the right-hand side is a form of seignorage revenue coming from the ability to sell liquid assets to the rest of the world.

Having provided a characterization of equilibrium conditions, we turn to analyze a steady state and, next, the effect of a tariff.

4 A WORLD DEBTOR WITH A PERMANENT TRADE DEFICIT

Assume that the endowments Y_{Ht} and Y_{Ft}^* are both growing at the constant rate g. Assume also that the domestic government chooses a constant tariff τ and lets the supply of bonds $\bar{B}_t$ grow at the rate g, keeping its debt to output ratio constant. For simplicity, we focus here on the case where the foreign tariff is zero. The case with a positive foreign tariff is in the appendix.

4.1 STATIONARY EQUILIBRIUM

Given these assumptions, there exists a stationary equilibrium in which the prices P_{Ht} and P_{Ft} are constant and all quantities—consumption levels, liquid and illiquid bond positions—grow at rate g in both countries.

To ensure that the Euler equations of both home and foreign consumers is satisfied the interest rate must be constant and equal to

$$i = \rho + \gamma g.$$

We need to make the assumption $\rho > (1 - \gamma) g$ to ensure that utility is well defined when consumption grows at rate g. This implies that the inequality $i > g$ holds in equilibrium.

We now adopt the following notational convention: for all quantities, lowercase variables denote detrended variables, for example,

$$c_t = C_t e^{-gt}.$$

In a stationary equilibrium, prices and detrended quantities are constant and we write them without a time subscript.

As the country net financial position is also growing at the rate g, the detrended trade deficit can be derived from (8) and written as

$$P_H d = (i - g)\, n + (i - i_b)\, b^*. \tag{9}$$

This equation has a natural interpretation. The term $(i - g)n$ on the right-hand side captures the flow revenue coming from the country net financial position, computed as if all net assets paid the return i. The second term $(i - i_b)b^*$ is the seignorage mentioned above and captures the flow of resources coming to the home country from providing liquidity services to the rest of the world.

Using the definition of d and assets market clearing, the foreign consumer's budget constraint can be rewritten as

$$P^* c^* = P_F y_F^* - P_H d. \tag{10}$$

The foreign demand for liquid bonds is similar to (7) and in stationary equilibrium takes the form

$$b^* = \psi^* \left(i - i_b\right)^{-\frac{1}{\gamma}} P^* c^*. \tag{11}$$

Combining (9)-(11) and rearranging we find the following expression for seignorage revenue

$$(i - i_b) b^* = \zeta \left(P_F y_F^* - (i - g)\, n\right)$$

where

$$\zeta = \frac{\psi^* \left(i - i_b\right)^{1-\frac{1}{\gamma}}}{1 + \psi^* \left(i - i_b\right)^{1-\frac{1}{\gamma}}} \in (0, 1).$$

Substituting back in (9) we can express the trade deficit as a function of the value of the foreign endowment and the net wealth of the home country:

$$d = (1 - \zeta)\, (i - g)\, \frac{n}{P_H} + \zeta \frac{P_F}{P_H} y_F^*. \tag{12}$$

The market clearing condition on the goods market, conditional on the trade deficit can be derived as in previous sections and can be written as

$$\mathcal{C}_H\left(P_F/P_H, d, \tau\right) + \mathcal{C}_H^*\left(P_F/P_H, d, \tau\right) = y_H, \tag{13}$$

where the definitions of the functions $\mathcal{C}_H$ and $\mathcal{C}_H^*$ are analogous to the ones in the two-period model.

4.2 Net financial position and trade deficit

Given initial asset positions A_0 and $B_0^* > 0$ and after choosing a numeraire, it is generally possible to find a stationary equilibrium, provided that some conditions are satisfied to ensure the solvency of both countries, i.e., provided that the net financial position N_0 is not too large or too small.

However, instead of proceeding from initial conditions for A_0 and B_0^*, in the following proposition we construct a family of equilibria by fixing the value of the liquidity premium, $\lambda = i - i_b > 0$, and the value of the Home net financial position expressed in Foreign goods, $\nu = n/P_F$. This approach to equilibrium construction will be useful to construct examples and for the exercises we will do later.

The proof of the proposition is in the Appendix.

Proposition 3 *Given any $\lambda > 0$ and any ν in some interval $[\underline{\nu}, \overline{\nu}]$, with $\underline{\nu} < 0 < \overline{\nu}$, there exists a stationary equilibrium in which the illiquid bond interest rate is $i = \rho + \gamma g$, the liquid bond interest rate is $i_b = i + \lambda$, good prices are constant, and all quantities grow at rate g.*

A corollary of this proposition is that we can construct examples of stationary equilibria with the following features.

Corollary 1 *Choosing a value of ν negative but not too far from zero, we obtain a stationary equilibrium in which the Home country:*

i. *Is a net debtor to the rest of the world, $N_t < 0$;*

ii. *Runs a permanent current account deficit, $\dot{N}_t < 0$;*

Iii. *Runs a permanent trade deficit*, $D_t > 0$.

The current account balance is just the change in the net foreign asset position $\dot{N}_t$. Therefore, point (ii) is an immediate implication of $\nu < 0$ and the fact that in a stationary equilibrium all asset positions grow at rate g so $\dot{N}_t = gN_t < 0$.

Point (iii) follow from equation (12) and the fact that, given the liquidity premium λ, we can compute ζ and choose a $\nu < 0$ so that

$$\frac{P_H}{P_F} d = (1-\zeta)(i-g)\nu + \zeta y_F^* > 0.$$

The interpretation of this equation is straightforward. If there is no world demand for liquidity and $\psi^* = 0$ then the coefficient $\zeta = 0$ and a permanent trade deficit can only be financed by the net returns on a positive net foreign asset position $\nu > 0$. However, with positive world liquidity demand we have $\zeta > 0$ and the the seignorage term $(i - i_b)b^*$ adds to the resources country Home can use for domestic spending.

Figure 3: Net worth and the trade deficit of the Home country in a stationary equilibrium.
Note: Both net worth and the trade deficit are measured in units of good F. The solid line uses $\zeta = 0.30$, $i - g = 0.05$, and $e_F^* = 0.5$, the dashed line has $\psi^* = 0$ and therefore $\zeta = 0$.

Figure 3 illustrates this point by plotting the simple linear relation between ν and the trade deficit expressed in foreign goods $(P_H/P_F)d$. We compare two economies, one with $\psi^* > 0$ and one with no liquidity demand from Foreign consumers, $\psi^* = 0$. The crucial observation is that the presence of world liquidity demand shifts up the level of trade deficits compatible with a given net foreign position.

5 THE EFFECTS OF A TARIFF SHOCK

Take the stationary economy of the last section and consider what happens if the domestic fiscal authority increases the tariff τ at $t = 0$. The change in τ is completely unexpected and permanent.

The response to the tariff shock is easy to derive thanks to the fact that the economy immediately jumps to a new stationary equilibrium. The only tricky step is to check what happens to asset positions at the moment of the shock. That adjustment depends on the unit of account in which the assets were originally denominated and on the assumptions we make on how the domestic government adjusts its liquidity supply. We consider two cases, the first is easier to analyze, the second is more realistic.

In both cases, we assume that the Home government adjusts the supply of bonds $\bar{B}$ to keep the liquidity premium constant. The assumption is an attempt to separate the implications of the tariff shock from the implications of changes in liquidity seignorage. We discuss the effect of alternative assumptions on bond supply in Section 5.4.

We use a minus subscript to denote values in the stationary equilibrium immediately before the shock, and a plus subscript to denote values immediately after the shock.

5.1 ALL ASSETS DENOMINATED IN F GOODS

Consider the case in which both liquid and illiquid asset are denominated in foreign goods. To do so, we simply choose good F as numeraire and set

$$P_F = 1.$$

In this case, we guess and verify that the detrended asset positions a and b^* remain unchanged after the shock.

Net wealth in nominal terms $N_t = A_t - B_t^*$ is unaffected by the shock, so $n_+ = n_-$. Given our assumption that the liquidity premium λ is kept constant, ζ is also constant, and equation (12) implies that the nominal value of the trade deficit $P_H d$ is unaffected.

Substituting $P_H d$ in (10) shows that foreign nominal consumption $P^* c^*$ is unaffected. Using this in equation (11) confirms our conjecture that foreign liquidity demand remains unchanged at the initial level: $b^*_+ = b^*_-$.

The demand for liquidity by domestic agents is in general affected by the tariff shock, but our assumption is that there is an instantaneous adjustment in the supply of bonds, which are transfered directly to domestic households, so that

$$\bar{B}_+ - \bar{B}_- = B_+ - B_-.$$

This transfer ensures that the market for liquid bonds continues to clear at the initial premium λ.

We then obtain the following neutrality result.

Proposition 4 *(Neutrality of trade balance) If the foreign good is the numeraire ($P_F = 1$) and the domestic government adjusts the supply of bonds to keep λ unchanged, an unexpected, permanent increase in τ leads to:*

i. *Unchanged asset positions a, b^*;*

ii. *Unchanged paths for the trade deficit and for the current account deficit of the home country.*

So far it looks like the tariff is having no effects, but things change once we turn to the goods market. The relative price P_H/P_F must be affected by the tariff shock. In particular, an increase in τ increases the domestic demand for the home good in the good market clearing condition (13). Since P_F remains equal to 1 by our choice of numeraire, it is easy to show that the price P_H needs to increase to clear the goods market.

Gross trade flows are affected and in particular it is possible to show that both domestic imports c_F and domestic exports decrease both in terms of quantities $y_H - c_H$ and in value $P_H(y_H - c_H)$. But this leaves the trade balance exactly unchanged as

$$c_F - P_H(y_H - c_H) = P_H d.$$

Finally, we look at the effects on domestic spending.

Nominal spending of the domestic consumer, net of taxes, is $P_H y_H + P_H d$ and it increases, because $P_H d$ is unchanged and P_H is higher. If the initial level of τ is close to 0 we can also show that total nominal spending increases, because:

$$Pc = P_H y_H + P_H d + T,$$

and near $\tau = 0$ the tariff revenue T is increasing in τ. Notice that the change in Pc changes the liquidity demand of domestic consumers, which is why, in general we need the domestic government to adjust the supply of liquid bonds. We summarize the trade side of our results in the following proposition.

Proposition 5 *If the Foreign good is the numeraire ($P_F = 1$), and the domestic government adjusts the supply of bonds to keep λ constant, an unexpected, permanent increase in τ leads to:*

i. *A permanent increase in the price of the Home good p_H;*

ii. *A reduction of equal value in Home imports and Home exports.*

iii. *A permanent increase in domestic spending Pc.*

This is an example of an economy in which the conventional view of a separation between the financial side and the trade side of our models, which we discussed in the Introduction, holds exactly.

The reason for this separation is more easily understood if we contrast this model to the model of Section 2. In the model in this section, trade flows are stationary, which means that the effect of the tariff on P_H is constant in the current period and in all future periods. Therefore, the tariff does not alter the wedge between the Home and Foreign real interest rates and does not affect intertemporal trade motives.

5.2 Home liabilities denominated in H

A more realistic assumption, if we want to capture the role of the U.S. as a world liquidity supplier, is to assume that the liquid bonds B_t are denominated in dollars. We do that in our model by simply choosing H as the numeraire:

$$P_H = 1.$$

We can also capture the idea that a substantial fraction of U.S. assets abroad are real assets in foreign countries, accumulated through foreign direct investment. This leads us to assume that the illiquid asset is denominated in F goods. Namely, let $\tilde{A}_t$ denote illiquid F-good-denominated assets held by Home consumers.

With these assumptions, the net foreign asset position of Home is now

$$N_t = A_t - B_t^* = P_{Ft}\tilde{A}_t - B_t^*.$$

The change in the price P_F at the moment of the shock has a discrete valuation effect on the country net foreign asset position, which, after the shock is

$$N_+ = P_{F+}\tilde{A}_- - \tilde{B}_-^*.$$

The stationary trade deficit after the tariff shock is then

$$d_+ = (1-\zeta)(i-g)\,n_+ + \zeta P_{F+}y_F^*. \tag{14}$$

As in the previous case, the tariff shock leads to an increase in the Home relative price P_H/P_F. However, now this is achieved by a reduction in P_F instead of an increase in P_H. This has a negative feedback effect on the trade deficit d.

Proposition 6 *If the Home good is the numeraire ($P_H = 1$), illiquid assets are denominated in good F, and the Home government adjusts the supply of bonds to keep λ constant, a permanent increase in τ leads to:*

i. *A permanent fall in the price of the Foreign good P_F;*

ii. *A reduction in Home net foreign asset position (denominated in H goods);*

iii. *A reduction in the Home trade deficit d.*

In this case, the separation between the financial and the trade side of the model is no longer present, but the channel of transmission is quite different from the model of Section 2. Now the effect operates through a valuation effect. A simple interpretation is that we achieve a real appreciation in the U.S. via a strengthening of the U.S. dollar. Given the mismatch between Home assets and liabilities, this lowers Home's net financial wealth and pushes toward a smaller trade deficit in the new stationary equilibrium.

At the same time, the fall in the price of the foreign good P_F reduces the value of Foreign total wealth in terms of the numeraire. As Foreign nominal spending $P^* c^*$ falls so does the demand for liquid bonds b^*. This reduction in Foreign liquidity demand lowers seignorage $(i - i_b)b^*$ earned by Home, further limiting its ability to finance the trade deficit. Finally, using the balance sheet constraint of Home in the new stationary equilibrium $n_+ = a_+ - b^*_+$, we see that the reduction in Home net wealth and the fall in liquidity demand forces the domestic economy to cut its holdings of illiquid assets a. This decline in gross positions reflects a loss in "privilege", as Home can no longer finance the initial level of foreign asset holdings.

5.3 Tariff war

The valuation effects behind Proposition 6 depend entirely on the response of relative prices. With a foreign tariff, everything is reversed.

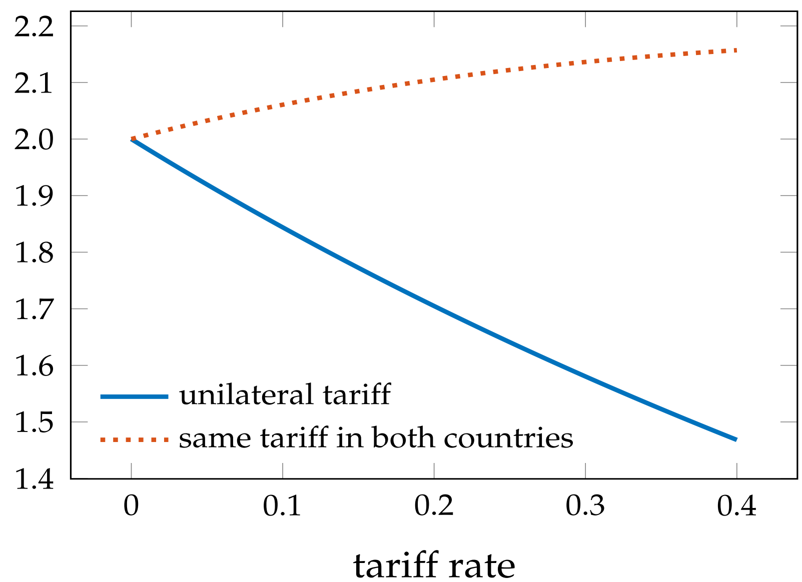
Proposition 7 *If the domestic government adjusts the supply of bonds to keep λ unchanged, an unexpected, permanent increase in the foreign tariff τ^* lowers Home terms of trade and increases the trade deficit as long as the initial tariff and the tariff shock are both small.*

Proof See Appendix G. ■

The intuition is straightforward. When the foreign government introduces a tariff, the terms of trade P_H/P_F fall in response to lower demand for Home goods. The valuation effects that previously drove the decrease in the trade deficit now work in the opposite direction. This is in contrast to the intertemporal model of Section 2, where a bilateral increase in tariffs strengthened the effect on Home's trade deficit. In that model, tariffs affected saving and investment decisions through their impact on intertemporal prices, not through valuation effects on existing asset positions. Here, the real interest rate channel is muted and the only way in which tariffs can affect the trade balance is through valuation effects.

This shows that if the foreign country retaliates, the resulting fall in the terms of trade will offset and may even undo the reduction in the trade deficit achieved by Home's tariff. Figure 4 illustrates this possibility. While a unilateral tariff reduces the trade deficit (blue line), bilateral tariffs increase the deficit above its initial level (orange line), reversing the effect of the domestic tariff.

Figure 4: Effect of unilateral and bilateral tariffs on the home country's trade deficit.



Note: Liquidity supply model with $\gamma = 2$, $\omega = 0.6$, and $\epsilon = 2$. We set $g = 0$, and normalize endowments to $y_H = 1$, $y_F = 1$. The liquidity premium is fixed at $\lambda = 0.02$ and ρ is chosen so that $i - g = 0.01$. The liquidity-demand parameters (ψ, ψ^*) and Home net worth are chosen to match the targets $d/e_H = 0.02$, $b/e_H = 0.5$, and $b^*/e_F = 1$.

5.4 Alternative assumptions on bond supply

Our results so far relied on the assumption that the domestic government stabilizes the liquidity premium when the tariff is introduced. We now briefly discuss what changes if the supply of liquid bonds is instead fixed.

With fixed supply, the liquidity premium λ must adjust to clear the market. When all assets are denominated in foreign goods, the tariff increases domestic demand for liquidity while leaving foreign demand unchanged, pushing the premium up. If $\gamma < 1$ the Laffer curve is downward sloping, so a higher premium tends to lower seigniorage and the trade deficit decreases. If instead $\gamma > 1$ the Laffer curve is upward sloping, and the trade deficit increases.

When Home liabilities are denominated in Home goods, the analysis is more subtle. The valuation effects from Section 5.2 lower both the trade deficit and foreign demand for liquidity, while domestic demand for liquidity rises. If the foreign response dominates, the global demand for liquidity falls, putting downward pressure on the premium. The effect on the trade deficit is then determined by the slope of the Laffer curve, as before.

This discussion shows that the mechanisms identified in the previous sections continue to play a key role. Tariffs affect the trade deficit through valuation effects and through changes in liquidity demand that affect seigniorage. Whether the government stabilizes the premium or lets it adjust governs the strength of these forces rather than introducing new ones.

6 HOME INCENTIVES TO DISTORT

We now turn to the welfare implications of tariffs. A recurring theme in what follows is the tension between reducing the trade deficit and improving domestic welfare. Tariffs can improve welfare through terms-of-trade effects, but the valuation effects that shrink the deficit also undermine their desirability. We first revisit the welfare effects of tariffs in our model. We then look at the possibility that tariffs reduce Foreign's appetite for Home liquidity. Finally, we consider a tax on foreign liquidity holdings (Miran 2024's *user fee*) as an alternative for reducing the trade deficit.

6.1 WELFARE EFFECTS OF TARIFFS

Standard trade theory suggests that, from a unilateral perspective, a large country can benefit from a tariff by improving its terms of trade. In our model, this logic continues to hold, but valuation effects weaken the marginal benefit of a tariff when Home liabilities are denominated in Home goods.

Consider first the case where all assets are denominated in Foreign goods. Here, the tariff improves Home terms of trade without generating any valuation effects on the net foreign asset position. The increase in P_H/P_F yields a positive income effect to the domestic consumer, who is a net seller of good H. It is therefore easy to show that near zero tariffs, a small increase in τ is always welfare improving for the domestic consumer. For small tariffs, this terms-of-trade gain dominates the distortionary cost, raising Home welfare. Naturally, for higher initial tariffs or for a large change in τ the distortionary effect can dominate and the tariff can be welfare reducing.

When Home liabilities are denominated in Home goods, valuation effects weaken the marginal welfare gain from small tariff increases. As we show in Section 5.2, a tariff leads to an appreciation of the Home currency that generates a negative valuation effect: the real value of Home's foreign assets (denominated in F) falls relative to its liabilities (denominated in H). This "capital loss" acts as a transfer to the rest of the world, counteracting the terms-of-trade gain. The same mechanism that allows the tariff to reduce the trade deficit also undermines its desirability from a welfare perspective.

Proposition 8 *Suppose the domestic government adjusts supply of bonds to keep λ unchanged. Starting from $\tau = 0$, a small increase in the Home tariff raises Home welfare when all assets are denominated in F goods. If instead Home has liabilities denominated in H goods, the same tariff increase delivers a smaller welfare gain due to valuation effects.*

PROOF See Appendix C. ∎

6.2 THE FRAGILITY OF PRIVILEGE

The analysis so far assumes that foreign demand for liquidity is not directly affected by Home's trade policy. But tariffs or other policy actions may erode foreign appetite for Home liabilities. We model this possibility as an unexpected, permanent drop in ψ^*, the parameter that governs the level of Foreign demand for liquid bonds.

This drop in liquidity demand reduces the trade deficit, but it leaves the Home consumer worse off. The mechanism is a classic transfer-problem effect: with lower foreign demand for Home liquidity, seigniorage falls, reducing the resources available to finance domestic spending. To satisfy its intertemporal budget constraint, Home must run a smaller trade deficit. This adjustment requires a real depreciation to increase exports and reduce imports, lowering the terms of trade and the real income of domestic consumers. Once again, a reduction in the trade deficit goes hand in hand with lower welfare.

Proposition 9 *Suppose Home is a net debtor in the initial stationary equilibrium. If the domestic government adjusts the supply of bonds to keep λ unchanged, an unexpected, permanent fall in foreign demand for liquid bonds (a small drop in ψ^*) reduces the trade deficit and makes Home consumers worse off, regardless of the unit of account in financial markets.*

PROOF See Appendix D. ∎

6.3 MIRAN'S DILEMMA

Suppose now that foreign demand for liquidity is stable and that tariffs are unable to reduce the trade deficit, as in Section 5.1. The domestic government may instead consider using a tax τ_b on foreign holdings of liquid bonds in order to reduce the trade deficit. This idea of introducing a "user fee" on foreign holdings of reserves was recently discussed by Miran (2024). It turns out that, through the lens of our model, such a policy cannot simultaneously improve domestic welfare and reduce the trade deficit. We call this result "Miran's dilemma".

Proposition 10 ("Miran's dilemma") *Suppose Home is a net debtor in the initial equilibrium. If all assets are denominated in foreign goods and the domestic government adjusts the supply of bonds to keep λ unchanged, a small tax on foreign liquidity holdings τ_b is either (i) welfare improving for Home and increases the trade deficit, or (ii) welfare reducing for Home and decreases the trade deficit.*

PROOF See Appendix E. ∎

The logic behind this result is not surprising. The introduction of the tax lowers the return on liquid bonds for foreigners. Depending on the elasticity of foreign demand for liquidity, this can either reduce or increase seigniorage. If the Laffer curve is upward sloping, Home is able to extract more resources from Foreign and domestic consumers are better off, but Home will end up running a larger trade deficit. If instead we are on the downward sloping part of the Laffer curve, the tax leads to a loss in seigniorage, which does reduce the trade deficit. However, this also reduces Home purchasing power and ends up lowering domestic welfare.

Across all three scenarios, the same tension emerges: policies that reduce the trade deficit tend to lower domestic welfare, while policies that raise welfare tend to widen the deficit.

7 Conclusion

This paper revisits the connection between tariffs and current account balances, highlighting how the nature of trade deficits shapes the effectiveness of tariff policy. In the standard intertemporal framework, higher tariffs increase the real interest rate in borrowing countries and reduce it in lending countries, thereby discouraging intertemporal trade and reducing current account imbalances. This mechanism works whether tariffs are imposed unilaterally or reciprocally, and is central to much of the existing literature.

This logic breaks down in a world where deficits are structural rather than transitory. When a country can run permanent deficits by supplying liquid assets to the world, the real interest rate channel is muted. Tariffs do not necessarily alter saving incentives, and their impact on the current account is limited to valuation effects at the moment of introduction. When all assets are denominated in the creditor's currency, tariffs have no effect on the trade deficit. With more realistic currency denomination, a permanent unilateral tariff can reduce the deficit, but this effect disappears if the foreign country retaliates.

Our analysis also reveals a tension between reducing trade deficits and improving domestic welfare. Policies that effectively reduce the deficit often come at the expense of domestic welfare. This tradeoff, what we call "Miran's dilemma," suggests that a reduction in the trade deficit may signal the loss of the country's exorbitant privilege.

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A Derivations for Section 2

$$\mathcal{C}_H(P_{Ht}/P_{Ft}, D_t, \tau) = \frac{\omega(P_{Ht}/P_{Ft})^{1-\epsilon}}{\omega(P_{Ht}/P_{Ft})^{1-\epsilon} + (1-\omega)\,(1+\tau_t)^{-\epsilon}}\,(Y_{Ht} + D_t)\,,$$

$$\mathcal{C}_H^*(P_{Ht}/P_{Ft}, D_t, \tau^*) = \frac{(1-\omega)(1+\tau_t^*)^{-\epsilon}(P_{Ht}/P_{Ft})^{1-\epsilon}}{\omega + (1-\omega)\,(1+\tau_t^*)^{-\epsilon}(P_{Ht}/P_{Ft})^{1-\epsilon}}\left(\frac{P_{Ft}}{P_{Ht}}Y_{F,t}^* - D_t\right).$$

A.1 Proof of Proposition 1

Let $\rho \equiv P_H/P_F$ denote the terms of trade. We define the home and foreign spending shares on the Home good as:

$$s_H(\rho,\tau) \equiv \frac{\omega\,\rho^{1-\epsilon}}{\omega\,\rho^{1-\epsilon} + (1-\omega)(1+\tau)^{-\epsilon}},$$

$$s_H^*(\rho,\tau^*) \equiv \frac{(1-\omega)(1+\tau^*)^{-\epsilon}\,\rho^{1-\epsilon}}{\omega + (1-\omega)(1+\tau^*)^{-\epsilon}\,\rho^{1-\epsilon}}.$$

With home bias ($\omega > 1/2$) and nonnegative tariffs, $s_H > s_H^*$, and we can write the demands from the main text as $C_H = s_H\,(Y_H + D)$ and $C_H^* = s_H^*\left(\frac{Y_F^*}{\rho} - D\right)$.

Differentiating the home demand functions $\mathcal{C}_H$ and $\mathcal{C}_H^*$ gives

$$\frac{\partial \ln C_H}{\partial \ln \rho} = -(\epsilon-1)(1-s_H), \qquad \frac{\partial \ln C_H}{\partial \ln(1+\tau)} = \epsilon(1-s_H), \qquad \frac{\partial \ln C_H}{\partial \ln D} = \frac{D}{Y_H + D},$$

$$\frac{\partial \ln C_H^*}{\partial \ln \rho} = -\Big[\epsilon - s_H^*(\epsilon-1)\Big] - \frac{\rho D}{Y_F^* - \rho D}, \qquad \frac{\partial \ln C_H^*}{\partial \ln D} = -\frac{\rho D}{Y_F^* - \rho D}.$$

Totally differentiating market clearing $C_H + C_H^* = Y_H$ and rearranging,

$$\frac{d\ln\rho}{d\ln(1+\tau)} = \frac{C_H(1-s_H)\,\epsilon}{C_H(1-s_H)(\epsilon-1) + C_H^*\Big[\epsilon - s_H^*(\epsilon-1) + \frac{\rho D}{Y_F^* - \rho D}\Big]} = \frac{\epsilon}{\epsilon - 1 + K},$$

where, after some rearranging,

$$K \equiv \frac{C_H^*}{C_H(1-s_H)}\left[(\epsilon-1)(1-s_H^*) + \frac{Y_F^*}{Y_F^* - \rho D}\right].$$

The elasticity of ρ with respect to the home tariff depends on D through K. In particular, the elasticity is increasing in D if and only if K is decreasing in D.

To prove that K is decreasing in D, let us rewrite it as

$$K = \frac{(\epsilon - 1)C_H^* + \frac{s_H^*}{1-s_H^*}\frac{Y_F^*}{\rho}}{\frac{1-s_H}{1-s_H^*}C_H}. \tag{15}$$

First, notice that $d\rho/dD > 0$ follows from total differentiation of home good market clearing condition and

$$C_H \frac{\partial \ln C_H}{\partial \ln D} + C_H^* \frac{\partial \ln C_H^*}{\partial \ln D} = \rho D \left(s_H - s_H^*\right) > 0,$$

It follows that $C_H^* = \mathcal{C}_H^*(\rho(D,\tau), D, \tau)$ is decreasing in D because $\mathcal{C}_H^*$ is decreasing in both ρ and D. From market clearing, $C_H = Y_H - C_H^*$ is increasing in D. Also, from $d\rho/dD > 0$ and $\epsilon \geq 1$, both s_H and s_H^* are decreasing in D and, from $s_H > s_H^*$, we have

$$\frac{d\ln(1-s_H)}{d\ln\rho} - \frac{d\ln(1-s_H^*)}{d\ln\rho} = (\epsilon - 1)(s_H - s_H^*) > 0.$$

Inspecting (15), we conclude that K is decreasing in D, which completes the proof.

A.2 Proposition 11: Exogenous Terms of Trade

Proposition 11 *If terms of trade are exogenous and constant over time, a permanent tariff has no effect on the equilibrium trade deficit.*

PROOF Suppose the terms of trade are exogenous and constant at $\bar{\rho} \equiv P_{Ht}/P_{Ft}$ and consider a permanent home tariff $\tau_t = \tau$. Using (4) and $P_{Ht} = \bar{\rho}\, P_{Ft}$, the home price index can be written as

$$P_t = P_{Ft}\,\phi(\bar{\rho},\tau), \qquad \phi(\bar{\rho},\tau) \equiv \left[\omega\bar{\rho}^{1-\epsilon} + (1-\omega)(1+\tau)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}},$$

where $\phi(\bar{\rho},\tau)$ is constant over time. Demands are

$$C_{Ht} = \omega\left(\frac{\bar{\rho}P_{Ft}}{P_t}\right)^{-\epsilon} C_t, \qquad C_{Ft} = (1-\omega)\left(\frac{(1+\tau)P_{Ft}}{P_t}\right)^{-\epsilon} C_t.$$

Plugging these into $P_{Ht}C_{Ht} + P_{Ft}C_{Ft} = P_{Ht}(Y_{Ht} + D_t)$, dividing by P_{Ft}, and rearranging yields

$$C_t = \frac{\bar{\rho}}{\phi(\bar{\rho},\tau)^{\epsilon}\left[\omega\bar{\rho}^{1-\epsilon} + (1-\omega)(1+\tau)^{-\epsilon}\right]}\,(Y_{Ht} + D_t).$$

The factor multiplying $(Y_{Ht} + D_t)$ is the same in $t = 1, 2$, so the ratio C_1/C_2 is independent of the tariff. Similarly, $P_{Ht}/P_t = \bar{\rho}/\phi(\bar{\rho}, \tau)$ is constant over time, while P_1/P_2 is independent of τ. Plugging these expressions into the Euler equation and the intertemporal budget constraint gives

$$(Y_{H1} + D_1)^{-\gamma} = \beta(1 + r_1)(Y_{H2} + D_2)^{-\gamma}, \qquad D_1 + \frac{1}{1 + r_1} D_2 = 0,$$

which coincides with the zero-tariff case. It follows that with exogenous terms of trade, a permanent tariff has no effect on the trade deficit. ■

A.3 Proof of Proposition 2

Fixing $\tau^* = 0$, define

$$F(D_1, D_2, \tau, \theta) \equiv \frac{1 + r_1}{1 + r_1^*} - \frac{P_1/P_2}{P_1^*/P_2^*},$$

$$G(D_1, D_2, \tau, \theta) \equiv D_1 + \frac{1}{1 + r_1} \frac{\frac{P_{H2}}{P_2}}{\frac{P_{H1}}{P_1}} D_2,$$

so equilibrium (D_1, D_2) satisfies $F = 0$ and $G = 0$.

Differentiating with respect to τ and eliminating $\partial D_2/\partial\tau$ gives

$$\frac{\partial D_1}{\partial \tau} = \frac{F_{D_2} G_\tau - F_\tau G_{D_2}}{F_{D_1} G_{D_2} - F_{D_2} G_{D_1}}. \tag{16}$$

Evaluating the partials at the symmetric allocation, defining $\Lambda \equiv 1 + 2(\epsilon - 1)\omega > 0$, and using $Y_{F1} = Y_{H1}$, $Y_{F2} = Y_{H2}$,

$$F_{D_1} = \frac{1 - 4(\omega - 1)\omega(-1 + \epsilon\gamma)}{(\omega - 1)\Lambda Y_{H1}}, \qquad F_{D_2} = \frac{-1 + 4(\omega - 1)\omega(-1 + \epsilon\gamma)}{(\omega - 1)\Lambda Y_{H2}},$$

$$G_{D_1} = 1, \qquad G_{D_2} = \frac{1}{1 + r_1} = \beta\left(\frac{Y_{H2}}{Y_{H1}}\right)^{\gamma} > 0.$$

For $\omega > 1/2$ and $\epsilon \geq 1$, $F_{D_1} < 0 < F_{D_2}$, so $F_{D_1} G_{D_2} - F_{D_2} G_{D_1} < 0$. By continuity, this sign remains unchanged in a neighbourhood of $(\tau, \theta) = (0, 0)$.

Near $(\tau, \theta) = (0, 0)$, using the fact that β does not enter the static equilibrium conditions directly (so $F_{\tau\beta} = F_{\tau\beta^*} = G_{\tau\beta} = G_{\tau\beta^*} = 0$) and that at the symmetric allocation $G_{\tau D_1} = G_{\tau D_2} = 0$, the numerator expands as

$$F_{D_2} G_\tau - F_\tau G_{D_2} = -\frac{\Phi_1 \omega}{(\omega - 1)\Lambda^2} G_{D_2} \left(\frac{D_1}{Y_{H1}} - \frac{D_2}{Y_{H2}}\right) + O(\theta^2),$$

with $\Phi_1 > 0$ for $\omega > 1/2$ and $\epsilon \geq 1$ (see below). Since $D_1 > 0$ and $D_2 < 0$ for $\theta > 0$, the bracket is positive. Since $\Phi_1 > 0$ and $\omega < 1$ ensures that $-\frac{\Phi_1 \omega}{(\omega-1)\Lambda^2} > 0$, the leading term in the numerator of (16) is positive, while the denominator is negative. Therefore $\partial D_1/\partial\tau < 0$ in a neighbourhood of $(\tau, \theta) = (0, 0)$.

AUXILIARY DERIVATIONS

Non-zero cross-partial derivatives used in the expansion of the numerator in (16) are

$$F_{\tau D_1} = \frac{\omega\,\Phi_1}{(\omega-1)\Lambda^2\,Y_{H1}}, \qquad F_{\tau D_2} = \frac{\omega\,\Phi_2}{(\omega-1)\Lambda^2\,Y_{H2}},$$

with

$$\begin{aligned}
\Phi_1 &= -(1-2\omega)^2(\omega-1) + \epsilon(\omega-1)(2\omega-1)\left[-\gamma + 2(1+\gamma)\omega\right] \\
&\quad + \epsilon^2\Big[-1 + 2\omega + \gamma\big(2 - 2\omega(2 + \omega(-3+2\omega))\big)\Big], \\
\Phi_2 &= (1-2\omega)^2(\omega-1) - \epsilon(\omega-1)(2\omega-1)\left[-\gamma + 2(1+\gamma)\omega\right] \\
&\quad + \epsilon^2\Big[1 - 2\omega - \gamma\big(2 - 2\omega(2 + \omega(-3+2\omega))\big)\Big].
\end{aligned}$$

Notice that $\Phi_2 = -\Phi_1$. We now show that under $\omega > 1/2$ and $\epsilon \geq 1$, $\Phi_1 > 0$. Write $\Phi_1 = A(\omega,\epsilon) + \gamma\,B(\omega,\epsilon)$ with

$$A(\omega,\epsilon) = (1-\omega)(1-2\omega)^2 - 2\epsilon\,\omega(1-\omega)(2\omega-1) + \epsilon^2(-1+2\omega),$$

$$B(\omega,\epsilon) = \epsilon(\omega-1)(2\omega-1)^2 + 2\epsilon^2\big(1 - 2\omega + 3\omega^2 - 2\omega^3\big).$$

Rewrite

$$A(\omega,\epsilon) = (2\omega-1)\left[\epsilon^2 - 2\epsilon\,\omega(1-\omega) + (1-\omega)(2\omega-1)\right].$$

For $\epsilon \geq 1$ the bracket is bounded below by its value at $\epsilon = 1$, namely $\omega > 0$, so $A > 0$ because $2\omega - 1 > 0$. Similarly,

$$B(\omega,\epsilon) = \epsilon(1-\omega)\Big[4\epsilon\omega^2 - 2\epsilon\omega + 2\epsilon - 4\omega^2 + 4\omega - 1\Big].$$

The bracket is increasing in ϵ. At $\epsilon = 1$ it equals $2\omega + 1 > 0$, so it is positive for all $\epsilon \geq 1$. With $1 - \omega > 0$, this implies $B > 0$. Thus both A and B are positive.

B Proof of Proposition 3

The value of $\zeta \in (0,1)$ is determined by λ. We can then summarize here the equilibrium conditions for a stationary equilibrium for a given ν:

$$d = \frac{P_F}{P_H}[(1-\zeta)(i-g)\nu + \zeta y_F^*],$$

$$\begin{aligned} c_H &= \frac{\omega (P_H/P_F)^{1-\epsilon}}{\omega (P_H/P_F)^{1-\epsilon} + (1-\omega)\,(1+\tau_t)^{-\epsilon}}\,(y_H + d)\,, \\ c_H^* &= \frac{(1-\omega)(1+\tau_t^*)^{-\epsilon}(P_H/P_F)^{1-\epsilon}}{\omega + (1-\omega)\,(1+\tau_t^*)^{-\epsilon}(P_H/P_F)^{1-\epsilon}} \left(\frac{P_F}{P_H} y_F^* - d\right), \\ y_H &= c_H + c_H^*. \end{aligned}$$

Substituting the first three conditions in the last (the market clearing condition), and using our assumption $\epsilon \geq 1$, it is easy to show that if $\nu = 0$ there exists a unique solution for ρ and, at the solution, both c_H and c_H^* are strictly positive. This follows because the economy is equivalent to a static economy in which the endowment of H is $(y_H, \zeta y_F^*)$ and the endoment of F is $(0, (1-\zeta) y_F^*)$.

By a continuity argument, these properties are preserved in a neighborhood of $\nu = 0$.

C Proof of Proposition 8

Consider the effect of a small, unexpected, permanent increase in the tariff τ, starting from $\tau = 0$. The domestic government adjusts the supply of bonds to keep the liquidity premium λ constant. Following the main text, we use a minus subscript to denote values in the stationary equilibrium immediately before the shock, and a plus subscript to denote values immediately after the shock. For any variable x, we denote the change $x_+ - x_-$ by dx.

When λ is unchanged, domestic welfare moves one for one with the consumption aggregate, so we look at how c responds to a small change in τ. By the envelope theorem,

$$\frac{dc}{d\tau} = \frac{\frac{\partial C}{\partial C_F}}{1+\tau}\left[(y_H - c_H)\,\frac{d\rho}{d\tau} + \frac{d(\rho d)}{d\tau} + \tau\,\frac{dc_F}{d\tau}\right], \tag{17}$$

where $\rho \equiv P_H/P_F$ and $\frac{\partial C}{\partial C_F} > 0$ is the partial derivative of the CES aggregator in (1). Evaluating (17) at $\tau = 0$ gives

$$\frac{dc}{d\tau}\Big|_{\tau=0} = \frac{\partial C}{\partial C_F}\left[(y_H - c_{H,-})\frac{d\rho}{d\tau}\Big|_{\tau=0} + \frac{d(\rho d)}{d\tau}\Big|_{\tau=0}\right], \tag{18}$$

We will keep using the spending shares s_H and s_H^* defined in Appendix A.1. For future reference, define

$$X_{H,-} \equiv \rho_-(y_H + d_-), \qquad X_{F,-} \equiv y_F^* - \rho_- d_-,$$

and

$$Z_- \equiv (1 - s_{H,-})y_H - \frac{1-\epsilon}{\rho}\left[X_{H,-}s_{H,-}(1-s_{H,-}) + X_{F,-}s_{H,-}^*(1-s_{H,-}^*)\right]. \tag{19}$$

Since $\epsilon \geq 1$, we have $Z_- > 0$.

All assets denominated in F goods. With $P_F = 1$, Proposition 4 implies $P_H d = \rho d$ is constant. So the sign of consumption response depends only on $d\rho/d\tau$. Totally differentiating the goods market clearing condition and evaluating at $\tau = 0$ gives:

$$\frac{d\rho}{d\tau}\Big|_{\tau=0} = \frac{\rho_-\,\epsilon(1-s_{H,-})c_{H,-}}{Z_-}. \tag{20}$$

Since $Z_- > 0$, $d\rho/d\tau > 0$ and welfare increases.

Home liabilities denominated in H goods. With predetermined real positions $\tilde{A}_-$ and $\tilde{B}_-^*$, condition (14) can be rewritten as:

$$\rho_+ d_+ = (1-\zeta)(i-g)\left(\tilde{A}_- - \rho_+\,\tilde{B}_-^*\right) + \zeta\, y_F^*.$$

Totally differentiating gives $d(\rho d) = -(1-\zeta)(i-g)\tilde{B}_-^*\, d\rho$. Substituting into (18):

$$\frac{dc}{d\tau}\Big|_{\tau=0} = \frac{\partial C}{\partial C_F}\left[(y_H - c_{H,-}) - (1-\zeta)(i-g)\tilde{B}_-^*\right]\frac{d\rho}{d\tau}\Big|_{\tau=0}. \tag{21}$$

The terms of trade response is now:

$$\frac{d\rho}{d\tau}\Big|_{\tau=0} = \frac{\rho_-\,\epsilon(1-s_{H,-})c_{H,-}}{Z_- + (1-\zeta)(i-g)\tilde{B}_-^*(s_{H,-} - s_{H,-}^*)}. \tag{22}$$

Since $s_{H,-} > s_{H,-}^*$, the denominator in (22) is larger than the denominator in (20). Valuation effects dampen the terms-of-trade improvement, and together with the direct effect in (21), they reduce the marginal welfare gain from raising τ, completing the proof.

D Proof of Proposition 9

Recall that ψ^* maps into the share $\zeta \in (0,1)$ that appears in equation (12). Since ζ is increasing in ψ^*, we treat the shock as an effective shock to ζ. We use a minus subscript to denote values immediately before the shock and a plus subscript to denote values immediately after the shock. For any variable x, we denote the change $x_+ - x_-$ by dx.

Home liabilities denominated in H goods. The condition for the trade deficit immediately after the shock can be rewritten as:

$$\rho_+ d_+ = (1-\zeta_+)(i-g)\left(\tilde{A}_- - \rho_+ \tilde{B}^*_-\right) + \zeta_+ y^*_F.$$

So $d(\rho d) = -(1-\zeta)(i-g)\tilde{B}^*_- \, d\rho + M_- \, d\zeta$, with $M_- \equiv y^*_F - (i-g)\left(\tilde{A}_- - \rho_- \tilde{B}^*_-\right)$. Totally differentiating the goods market clearing condition gives

$$0 = Z_- \, d\rho - \left(s_{H,-} - s^*_{H,-}\right) \, d(\rho d),$$

where Z_- is defined in (19) and the shares s_H and s^*_H are defined in Appendix A.1.

Solving the system, we get

$$\begin{aligned}
\frac{dd}{d\zeta} &= \frac{Z_- - d_-(s_{H,-} - s^*_{H,-})}{\rho_- \left[Z_- + (1-\zeta)(i-g)\tilde{B}^*_-(s_H - s^*_H)\right]} M_-, \\
\frac{d\rho}{d\zeta} &= \frac{(s_{H,-} - s^*_{H,-})}{Z_- + (1-\zeta)(i-g)\tilde{B}^*_-(s_{H,-} - s^*_{H,-})} M_-.
\end{aligned}$$

Since Home is a net debtor, $M_- > 0$. With $Z_- > 0$ and

$$\begin{aligned}
Z_- - d_-(s_{H,-} - s^*_{H,-}) = \frac{1}{\rho_-}\Big[& s^*_{H,-}(1-s_{H,-})X_{H,-} + s^*_{H,-}(1-s^*_{H,-})X_{F,-} \\
& + (\epsilon - 1)\big(X_{H,-}s_{H,-}(1-s_{H,-}) + X_{F,-}s^*_{H,-}(1-s^*_{H,-})\big)\Big] > 0,
\end{aligned}$$

both derivatives are positive, and a fall in ζ (that is, a fall in ψ^*), lowers both the trade deficit d and the terms of trade ρ.

Turning to welfare, using the same envelope argument used in Appendix C gives

$$dc = \frac{\frac{\partial C}{\partial C_F}}{1+\tau}\Big[(y_H - c_{H,-})\, d\rho + d(\rho d) + \tau \, dc_F\Big],$$

with $dc_F = \frac{1-s_{H,-}}{\rho}\left[\epsilon s_{H,-} X_{H,-} + s^*_{H,-} X_{F,-}\right] d\rho + (1 - s_{H,-}) d\,(\rho d)$. Since $d\rho < 0$ and $d(\rho d) < 0$ after a fall in ζ, we have $dc_F < 0$ and $dc < 0$. Recalling that domestic welfare is proportional to c, establishes that Home consumers are worse off.

All assets denominated in F goods. With everything in F units there are no valuation effects, so the formulas above become

$$\frac{dd}{d\zeta} = \frac{Z_- - d_-(s_{H,-} - s^*_{H,-})}{\rho_-\, Z_-}\,\tilde{M}_-,$$
$$\frac{d\rho}{d\zeta} = \frac{(s_{H,-} - s^*_{H,-})}{Z_-}\,\tilde{M}_-,$$

with $\tilde{M}_- = y^*_F - (i-g)(\tilde{A}_- - \tilde{B}^*_-)$. The same algebra shows $dd/d\zeta > 0$ and $d\rho/d\zeta > 0$ whenever Home is a net debtor. Therefore a fall in ζ leads to a reduction in the trade deficit and the terms of trade, and welfare falls by the same argument.

E PROOF OF PROPOSITION 10

Suppose all assets are denominated in F goods (no valuation effects) and assume the Home government adjusts the supply of liquid bonds to keep λ fixed. As in the main text, we use a minus subscript to denote values immediately before the shock and a plus subscript to denote values immediately after the shock. For any variable x, we denote the change $x_+ - x_-$ by dx.

A small tax on foreign liquidity holdings $\tau_b > 0$ raises the wedge faced by foreign holders to $\lambda_F \equiv \lambda + \tau_b$ without changing the premium for home consumers. The change in ζ is

$$d\zeta = (1-\zeta_-)\left(1 - \frac{1}{\gamma}\right)\frac{d\tau_b}{\lambda_{F,-}}.$$

Using (12) and definining $\tilde{M}_- \equiv y^*_F - (i-g)(\tilde{A}_- - \tilde{B}^*_-)$, we have

$$d(\rho d) = \tilde{M}_-\,(1-\zeta_-)\left(1 - \frac{1}{\gamma}\right)\frac{d\tau_b}{\lambda_{F,-}},$$

so $d(\rho d)$ has the sign of $\left(1 - \frac{1}{\gamma}\right) d\tau_b$ because $\tilde{M}_- > 0$ for a net debtor.

Totally differentiating market clearing for the home good gives

$$0 = Z_-\,d\rho - (s_{H,-} - s^*_{H,-})\,d(\rho d),$$

where $s_{H,-}$ and $s^*_{H,-}$ are the spending shares from Appendix A.1, and $Z_- > 0$ is defined in (19). So $d\rho$ shares the sign of $d(\rho d)$. From $d(\rho d) = \rho\, dd + d\, d\rho$,

$$dd = \frac{Z_- - d_-(s_{H,-} - s^*_{H,-})}{\rho Z_-}\, d(\rho d).$$

The numerator of this expression is positive by the same argument used in Appendix D, so dd shares the sign of $d(\rho d)$.

The envelope argument used in Appendix D implies

$$dc = \frac{\partial C}{\partial C_F}\frac{1}{1+\tau}\Big[(y_H - c_{H,-})\, d\rho + d(\rho d) + \tau\, dc_F\Big],$$

where $dc_F = \frac{1-s_{H,-}}{\rho}\left[\epsilon s_{H,-} X_{H,-} + s^*_{H,-} X_{F,-}\right] d\rho + (1 - s_{H,-})\, d(\rho d)$. As $d\rho$, $d(\rho d)$, and dc_F all share the sign of $(1 - \frac{1}{\gamma}) d\tau_b$, and the coefficients multiplying them are positive for $\tau \geq 0$, dc shares that sign for any initial tariff.

Since domestic welfare is proportional to c, welfare and the trade deficit move in the same direction as $(1 - \frac{1}{\gamma}) d\tau_b$. In particular, for $\gamma > 1$, a small tax on foreign liquidity holdings increases both Home welfare and the trade deficit, while for $\gamma < 1$ both decrease, completing the proof.

F STATIONARY EQUILIBRIUM WITH POSITIVE FOREIGN TARIFF

Let $\rho \equiv P_H/P_F$ denote the terms of trade. With a foreign tariff τ^* on imports of the H good, the foreign consumer's budget constraint becomes

$$(1+\tau^*)P_H c^*_H + P_F c^*_F + g(b^* - a^*) = P_F y^*_F + i a^* + i_b b^* + T^*_t.$$

Combining this with the foreign government's budget constraint, asset-market clearing, and the definition of d delivers

$$P_H c^*_H + P_F c^*_F = P_F y^*_F - P_H d. \tag{23}$$

Using (9) and the foreign demand for liquid bonds,

$$(i - i_b) b^* = \zeta\Big(P_F y^*_F - (i-g)n + \tau^* P_H c^*_H\Big). \tag{24}$$

Substituting (24) back into (9),

$$P_H d = (1-\zeta)(i-g)n + \zeta\left(P_F y_F^* + \tau^* P_H c_H^*\right). \tag{25}$$

Demand for the H good by foreigners can be written as $c_H^* = s_H^*(\rho, \tau^*)\left(\frac{y_F^*}{\rho} - d\right)$ with

$$s_H^*(\rho, \tau^*) \equiv \frac{(1-\omega)(1+\tau^*)^{-\epsilon}\rho^{1-\epsilon}}{\omega + (1-\omega)(1+\tau^*)^{-\epsilon}\rho^{1-\epsilon}}.$$

Plugging this into (25) and rearranging yields

$$d = \left[1 - \tilde{\zeta}(\rho, \tau^*)\right](i-g)\frac{\nu}{\rho} + \tilde{\zeta}(\rho, \tau^*)\frac{y_F^*}{\rho}, \tag{26}$$

where $\nu = \frac{n}{P_F}$ is Home net worth in units of F goods, and

$$\tilde{\zeta}(\rho, \tau^*) \equiv \zeta\,\frac{1 + \tau^* s_H^*(\rho, \tau^*)}{1 + \zeta\,\tau^* s_H^*(\rho, \tau^*)} \in (0, 1).$$

Equation (26), together with goods market clearing $c_H + c_H^* = y_H$, pins down (d, ρ) in the stationary equilibrium with a positive foreign tariff. As long as τ^* is not too large, the same continuity argument as in Proposition 3 guarantees existence of a stationary equilibrium with a permanent trade deficit.

G Proof of Proposition 7

Fix the domestic tariff τ and consider a small, permanent change in τ^* around $\tau^* = 0$.

We begin with the case where Home liabilities are denominated in F goods. Net worth in F goods immediately after the shock is $\tilde{A}_- - \rho_+ \tilde{B}_-^*$. Using (26), the condition for the trade deficit immediately after the shock becomes

$$\rho_+ d_+ = (1-\zeta)(i-g)\left(\tilde{A}_- - \rho_+ \tilde{B}_-^*\right) + \zeta y_F^* + \zeta \tau^* \rho_+ c_{H,+}^*. \tag{27}$$

Linearizing around $\tau^* = 0$:

$$d(\rho d) = -(1-\zeta)(i-g)\tilde{B}_-^*\, d\rho + \zeta s_{H,-}^* X_{F,-}\, d\tau^*. \tag{28}$$

The linearized market clearing condition for the home good is:

$$0 = Z_-\, d\rho + \epsilon s_{H,-}^*(1 - s_{H,-}^*)c_{H,-}^*\, d\tau^* - \left(s_{H,-} - s_{H,-}^*\right)\, d(\rho d), \tag{29}$$

where the shares are the same as in Appendix A.1 and Z is defined in (19). Solving for the terms of trade response:

$$\frac{d\rho}{d\tau^*} = \frac{\rho_- s^*_{H,-} c^*_{H,-} \left[\zeta\left(s_{H,-} - s^*_{H,-}\right) - \epsilon(1 - s^*_{H,-})\right]}{Z_- + (1-\zeta)(i-g)\tilde{B}^*_- \left(s_{H,-} - s^*_{H,-}\right)}. \tag{30}$$

Since $\zeta < 1 \leq \epsilon$ and $s_{H,-} - s^*_{H,-} \leq 1 - s^*_{H,-}$, the numerator is negative, so $d\rho/d\tau^* < 0$. Substituting back into (28), we find $d(\rho d)/d\tau^* > 0$. Finally, $dd/d\tau^* > 0$ follows from $d(\rho d) = \rho\, dd + d\, d\rho$.

When all assets are denominated in F goods, there are no valuation effects, so we can simply set $\tilde{B}^*_- = 0$ in the expressions above. The same arguments deliver $d\rho/d\tau^* < 0$ and $dd/d\tau^* > 0$, completing the proof.